MaxDEA Manual

Version 5.2

Powerful free DEA software: MaxDEA Basic

and

More powerful and professional DEA software: MaxDEA Pro

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Chapter 1: Main Features of MaxDEA

1.1 Main Features

MaxDEA is an easy-to-use but most powerful software for Data Envelopment Analysis, with the following attractive features:

- ✧ Most comprehensive DEA models and **all their possible combinations** (over 30,000).
- ✧ No limitation on the number of DMUs, inputs and outputs.
- ✧ DEA linear programming can be exported to text files of 3 formats: matrix format (*.txt), lp format (*.lp) and mps format (*.mps).
- ✧ Both fixed and portable licences (USB key) are available.
- ✧ Update to a higher version is free of charge.
- ✧ Easy to use. MaxDEA doesn't need installation, and it has user-friendly interface.
    - MaxDEA is a green software. MaxDEA needn't installation.
    - MaxDEA is very easy to backup. The program and the database for the DEA model are integrated into one access database file (.mdb, the only file needed for the program), which is very convenient to backup. The database and all options for the DEA model are saved permanently in the file, which means that after closing and reopening the file, the database and model specifications are still there unchanged.
    - MaxDEA uses standard database format, which means that you needn't to indicate the property of inputs/outputs with field names.
    - Multiple models can be run at the same time. Because the only files needed for running the program is MaxDEA.mdb, you can rename or copy this file freely. Each copy of this file contains one DEA model with all its data and specifications saved in the file. And you can open and run multiple files simultaneously.

1.2 Models in MaxDEA

MaxDEA has most comprehensive DEA models and all their possible combinations. The total number of the model combinations is over 30,000.

1) Radial, Non-radial (SBM) and hybrid (mixture of radial and Non-radial) models
2) Super-efficiency model
3) Malmquist model (three reference types: Adjacent, Fixed and Global)
4) Window model
5) Directional Distance Function Model
6) Cluster model
7) Nondiscretionary input/output model (including non-controllable model and measure specific model)
8) Bounded input/output model
9) Undesirable output model (including directional distance function model and inseparable good and bad outputs model)
10) Preference (weighted) model
11) Weak disposability model
12) FDH model
13) Restricted multiplier model (assurance region model, trade-offs between inputs and outputs)
14) Customized reference ret model (including variable-benchmark model, fixed-benchmark model and minimum efficiency model)
15) "Two stage" method for computing input/output weights in multiplier models. It is especially useful for computing cross efficiencies since there usually

exist multiple optimal solutions in multiplier models.

16) Cross efficiency model.

17) Network DEA model (With MaxDEA you can design and analyze the Network DEA models combined with all other envelopment models).

18) Dynamic model

and

19) All the possible combinations of the above models

(See 3.3.1.1, 3.3.2.1 for details.)

MaxDEA provides extensive information about the evaluated DMUs, including

1) Technical efficiency and its components
2) Scale efficiency
3) Super Efficiency
4) Cost efficiency
5) Revenue efficiency
6) Profit efficiency
7) Revenue/cost ratio efficiency
8) Input and output improvement targets
9) Returns to scale
10) Times of a DMU as a benchmark for another DMU
11) Malmquist productivity index
12) Efficiency change over time
13) Technological change over time
14) Cross efficiency

15) Allocative efficiency = Cost (or Revenue or Profit) efficiency / Technical efficiency

16) Dual Solution and Sensitivity Analysis.

(See 3.3.1.2 and 3.3.2.2 for details)

1.3 What's NEW

MaxDEA 5.2

- ✧ Two methods available to compute productivity changes in Malmquist model. One is the traditional multiplicative method that measure the relative change of productivity, and the newly added one is the additive method that measures the absolute change of productivity.

MaxDEA 5.1

- ✧ Support indicators (inputs and outputs) with negative values.

New in version 5.0

- ✧ Completely support for Directional Distance Function Model. MaxDEA 5.0 provides a unified method to compute the efficiency score for directional distance function Model. Directional distance function model is a generalized form of radial model.
- ✧ Three types of reference for Malmquist Model: Adjacent, Fixed and Global. Both the new types (Fixed and Global) of Malmquist indices are circular. And the Global Malmquist model doesn't suffer from the infeasibility problem.
- ✧ Dual Solution and Sensitivity Analysis, including Dual prices and their Sensitivity Analysis for envelopment models, benchmarks with values of lambda and projections for multiplier models, and Sensitivity Analysis of Objective function.
- ✧ Interface improved. Cost/Revenue/Profit and FDH models are moved from advanced models to basic models. The number of the combinations of basic models is over 200.

New in version 4.4

- ✧ MaxDEA uses the method developed by Maniadakis and Thanassoulis to compute Cost Malmquist Model, and uses similar methods to compute Revenue, Profit, and Revenue Cost Ratio Malmquist Index
- ✧ The option "α = β" is added to Radial Non-oriented models.

New in version 4.3

- ✧ The results of Scale Efficiency are separated from other results. To avoid misunderstanding of the scale efficiency and the scale effect in Malmquist models, the results of scale efficiency are provided separately. The option is moved from the tab of "Results" to "RTS" in the tab of "Basic Models".
- ✧ The scale effect in Malmquist model is decomposed into two parts: one is scale effect on efficiency change and the other is scale effect on technological change.

New in version 4.2

- ✧ Inseparable good and bad outputs model is added.
- ✧ Undesirable model can be combined with weak disposability model.
- ✧ Non-discretionary and bounded models can be combined with weak disposability model.

New in version 4.1

- ✧ **Portable licence** is available. This licence type uses a flash disk as the USB key. The license holder may work with MaxDEA on any computer the USB key is plugged into. This licence allows for extreme flexibility.
- ✧ Two additional LP formats are available: one is mps format, which is supported by most solvers; and the other is lp format, which is similar to the mathematical formulation.

- ✧ Variables (columns) and constraints (rows) are named according to their meanings in the exported LPs.
- ✧ When the number of the columns of the results is over 255, which MS Access does not support, the results will be exported to a comma delimited text file (*.csv), which can be opened by text editor, statistic software, or Excel 2007.
- ✧ Bugs fixed or improvements made for: 1) Super Revenue/Profit/Revenue Cost Ratio Models; 2) Cost/Revenue/Profit/RevenueCost Ratio Network Models with Nondiscretionary or Bounded inputs/outputs/intermediates; and 3) Super Network model with VRS.

New in version 4.0

- ✧ Model Orientation is extended from 3 types to 8 types.
- ✧ Linear programming equations of DEA models can be exported to text files. (details in *MaxDEA Linear Programming Manual*)
- ✧ Dynamic model is added.
- ✧ User interface is improved.

New in version 3.2

- ✧ Nondiscretionary and bounded options can be applied to indirect inputs/outputs (intermediate) in Network DEA models.
- ✧ Prices of non-discretionary and bounded inputs/outputs are optional in non-discretionary and bounded Cost/Revenue/Profit/Revenue Cost Ratio models.
- ✧ Prices of non-discretionary and bounded inputs/outputs are optional in non-discretionary and bounded Cost/Revenue/Profit/Revenue Cost Ratio

models.

✧ “Profit Ratio” model was renamed as “Revenue/Cost Ratio” model.

New in version 3.1

✧ Cluster model was redesigned. There are four types of cluster models: self-benchmarking, cross-benchmarking, downward-benchmarking and upward-benchmarking.

✧ A bug fixed for Radial and Hybrid Network DEA models.

New in version 3.0

✧ The only file needed for running the program is MaxDEA.mdb, which is further convenient to run and backup your DEA models.

✧ User interface is improved.

✧ Cluster model is added. Cluster model deals with the situation that the DMUs are categorized according to their characteristics. There are four types of cluster models: self-benchmarking, cross-benchmarking, downward-benchmarking and upward-benchmarking.

✧ Window model is added. Both balanced and unbalance panel data can be analyzed.

New in earlier versions

✧ MaxDEA 2.7 added cross efficiency models.

✧ MaxDEA 2.6 added the "two stage" method for computing input/output weights in multiplier models.

✧ MaxDEA 2.5 added Network DEA.

✧ MaxDEA 2.4 added the customized benchmarking model (including

variable-benchmark and fixed-benchmark).

- ✧ MaxDEA 2.3 added directional distance function model for undesirable outputs.
- ✧ MaxDEA 2.2 added Malmquist for multiplier models.
- ✧ MaxDEA 2.0 added Cost, Revenue, Profit and Revenue/Cost Ratio models, and undesirable output model.
- ✧ MyDEA 1.0 (renamed as MaxDEA from 2.0)
  - Distance: Radial, Non-radial (SBM) and Hybrid;
  - Orientation: Input-, Output- and Non- oriented;
  - RTS: CRS, VRS, NIRS, NDRS and GRS;
  - FDH model;
  - Super-efficiency;
  - Nondiscretionary model;
  - Bounded model;
  - Preference (weighted) model;
  - Malmquist model.

1.4 Compare MaxDEA Basic and MaxDEA Pro

MaxDEA Basic is powerful and free, and MaxDEA Pro is more powerful and professional.

Features		MaxDEA Basic	MaxDEA Pro
Number of DMUs		Not limited	Not limited
Basic models	All combinations of basic options, including	√	√
	Radial (CCR, BCC)	√	√
	Nonradial (SBM)	√	√
	Directional Distance Function	√	√
	FDH	√	√
	CRS, VRS, NIRS, NDRS, GRS	√	√
	Cost, Revenue, Profit	√	√
	Input-, Output-, and non-oriented	√	√
Advanced models	All combinations of advanced options, including		√
	Malmquist		√
	Window		√
	Dynamic		√
	Cluster and customized benchmarking		√
	Network		√
	Super-efficiency		√
	Nondiscretionary/Uncontrollable		√
	Bounded		√
	Undesirable		√
	Weak disposability		√
	Preference(weighted)		√
	Restricted multiplier		√
	Cross-efficiency		√
Export LPs			√
Number of Models		200^{+}	30000+

Chapter 2: System Requirements

MaxDEA runs under any windows systems in any language, including:

Windows 2000

Windows XP

Windows 2003

Windows Vista

Windows 7

MaxDEA is developed with VBA for Access, so Microsoft **Access** is required.

If the program file (MaxDEA.mdb) cannot open, it indicates that Microsoft Office Access is not installed in your computer, and you must install MS Access 2003, 2007, or 2010 (Professional edition **OR** Runtime edition) first.

Access 2010 Runtime can be downloaded **free** at Microsoft website:

http://www.microsoft.com/en-us/download/details.aspx?id=10910

Access 2007 Runtime can be downloaded **free** at Microsoft website:

http://www.microsoft.com/en-us/download/details.aspx?id=4438

Chapter 3: Step-by-step Guide

MaxDEA is easy to use, with the following successive steps:

1) import data, 2) define data and 3) set and run the model.

Note: Just skip the contents irrelevant to your DEA model.

3.1 Import Data

Menu： File - Import Data from File

3.1.1 Data format

The first step to develop a DEA model is data importation. Data types supported include: **Excel, Access, dBase and comma delimited text file**. The data format is standard without special requirements for field names and their orders. The field names of inputs/outputs can be named freely as you want.

Here, we give an example in the popular Excel format,

Table 3-1 Example for data format

DMU	Input1	Input2	Output1	Output2	OTHERS
A	4323	875	93608	187196	20
B	2295	469	225559	451099	10
C	6379	1286	327068	654116	20
D	6644	1339	201354	402688	30
E	1436	297	188926	377833	28
F	6281	1266	413738	827456	40
G	7459	1502	114022	228024	22

The first row of the Excel sheet must contain column names, and there must **not** be spaces **left** to the column names. For example, column names such as “ Input1” will result in an error. But spaces in the middle of column names or right to the column names, such as “Input1 ” and “Input 1”, are permitted, but not

recommended.

Note: If the data are imported from Excel, please confirm that there are no spaces between numbers in the Excel sheet, such as “1. 23” or “1 .23”. Such a problem often takes place if the data in Excel sheet are copied from PDF files.

3.1.2 DMU Name

DMU Name is the identifier for each DMU, so it must be **unique**. The name for DMUs can be characters or numbers. The results will be sorted by DMU name, so if you expect the results to be sorted by original order, we suggest that you name the DMUs as something like DUM_01, DMU_02, DUM_03, …….

3.1.3 Panel Data

For **panel data**, there must be an additional column indicating the periods. The data can be sorted by DMU name or by period, but not necessary. DMU Name must be **unique** within each period. Panel data can be used for panel-data models only, such as Malmquist, Window, Dynamic models.

Panel data can be sorted by period, like

Table 3-2(A) Example for panel data

Period	DMU	Input1	Input2	Output1	Output2
1	A	4323	875	93608	187196
1	B	2295	469	225559	451099
1	C	6379	1286	327068	654116
2	A	6644	1339	201354	402688
2	B	1436	297	188926	377833
2	C	6281	1266	413738	827456
3	A	7459	1502	114022	228024
3	B	4464	903	212444	424867
3	C	4524	915	462677	925334

Or sorted by DMU name, like

Table 3-2(B) Example for panel data

Period	DMU	Input1	Input2	Output1	Output2
1	A	4323	875	93608	187196
2	A	6644	1339	201354	402688
3	A	7459	1502	114022	228024
1	B	2295	469	225559	451099
2	B	1436	297	188926	377833
3	B	4464	903	212444	424867
1	C	6379	1286	327068	654116
2	C	6281	1266	413738	827456
3	C	4524	915	462677	925334

Or not sorted at all, like

Table 3-2(C) Example for panel data

Period	DMU	Input1	Input2	Output1	Output2
1	A	4323	875	93608	187196
2	B	1436	297	188926	377833
1	B	2295	469	225559	451099
3	A	7459	1502	114022	228024
3	B	4464	903	212444	424867
1	C	6379	1286	327068	654116
2	A	6644	1339	201354	402688
2	C	6281	1266	413738	827456
3	C	4524	915	462677	925334

The **period field** must be integer, such as

1, 2, 3……

2001, 2002, 2003……

But needn’t to be continuous. The following time series are permitted:

1, 2, 5, 8……

2001, 2005, 2009……

200101, 200102, 200302……

3.1.4 Cluster Data

For **cluster data**, there must be an additional column indicating the clusters. Each DMU belongs to only one cluster, and a DMU should not belong to different clusters.

The following are examples of cluster data:

Table 3-3(A) Example for data with clusters

DMU	Cluster	Input1	Input2	Output1	Output2
A	1	4323	875	93608	187196
B	1	2295	469	225559	451099
C	1	6379	1286	327068	654116
D	1	6644	1339	201354	402688
E	1	1436	297	188926	377833
F	1	6281	1266	413738	827456
G	2	7459	1502	114022	228024
H	2	4464	903	212444	424867
I	2	4524	915	462677	925334

Cluster data needn't be balanced. The number of DMUs in each cluster can be different.

Table 3-3(B) Example for panel data with clusters

Period	DMU	Cluster	Input1	Input2	Output1	Output2
1	A	1	4323	875	93608	187196
1	B	1	6644	1339	201354	402688
1	C	1	7459	1502	114022	228024
1	D	1	2295	469	225559	451099
1	E	1	1436	297	188926	377833
1	F	1	4464	903	212444	424867
1	G	2	6379	1286	327068	654116
1	H	2	6281	1266	413738	827456
1	I	2	4524	915	462677	925334
2	A	1	4341	944	93656	187279
2	B	1	6689	1374	201442	402767
2	C	1	7559	1511	114107	228101

2	**D**	**1**	2362	530	225604	451107
2	**E**	**1**	1513	381	188937	377886
2	**F**	**1**	4544	937	212446	424869
2	**G**	**2**	6406	1361	327102	654188
2	**H**	**2**	6369	1327	413797	827479
2	**I**	**2**	4563	942	462739	925393

Note: For panel data, each DMU must belong to the **same cluster** in different periods. For example, DMU A belongs to cluster 1 in period 1, and it must belong to cluster 1 in period 2 and other periods.

The **cluster field** must be integer, such as

1, 2, 3……

But they needn't to be continuous. The following time series are permitted:

1, 2, 5, 8……

2001, 2005, 2009……

200101, 200102, 200302……

Once is enough, after closing the program, the imported data will **not** be lost.

3.2 Define Data

Menu：Data - Define Data

The second step is to define data, indicating which column is the DMU name, which columns are inputs, and which columns are outputs.

After data importation, the “Data Define” window will open automatically. The fields for **DMU name**, **Inputs** and **Outputs** must be defined。

Important Notes:

1) The fields to be used for lower and upper bounds in bounded model, for direction vector in directional distance function model, and for prices in cost, revenue and profit models, must be kept ”Not defined”.

2) The fields to be used for indirect inputs/outputs, i.e. intermediate inputs/outputs, in Network and Dynamic models, must be defined as “Intermediate”.

3) “Cluster” is used to indicate the cluster ID in cluster models. Cluster ID must be integer.

If you want to eliminate some inputs or outputs from the model temporarily, just deactivate the check box. Only those active inputs/outputs will be included in the DEA model. The status of check box has no effect for DMU name field and period field.

Once is enough for data definition as well**.** After closing the program, the definition will **not** be lost.

3.3 Set and Run Model

Menu: Model - Run Envelopment Model

Model - Run Multiplier Model

Model -Express to Basic Models

The last step is to set the specifications and options for the DEA model. The menu "Model - Run Envelopment Model" is for envelopment models, "Model - Run Multiplier Model" is for multiplier models, and "Model -Express to Basic Models" is an express way for basic models (CCR and BCC), which is easier to use for new users.

After closing the program, all the options will **not** be lost. If your want to save a backup of the model, just copy the "mdb" file and rename it as you want, such as Model1.mdb, Model2.mdb.

3.3.1 Envelopment model

3.3.1.1 Model specifications

The specifications for envelopment models include basic specifications (Distance, Orientation and RTS) advanced specifications.

To run a DEA model, you should set the basic specifications (Basic Model) first, and then set the extended specifications(Advanced Model). If the extended specifications under the basic specifications are not available, the extended specification's check box will be disabled (the color will turn grey). This change is refreshed after every operation.

3.3.1.1.1 Basic Specifications(Basic Model)

Basic specifications for envelopment models include distance, orientation and returns to scale.

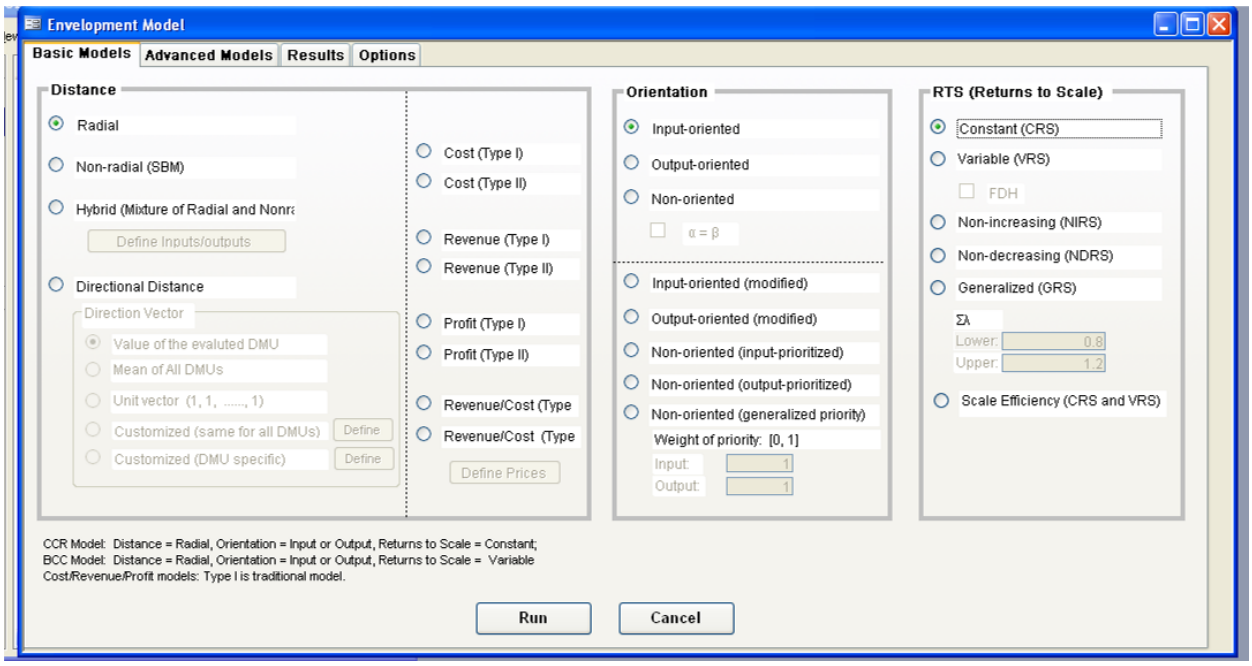


Figure 3-1-A Basic specifications for Envelopment model

Distance

Distance functions to calculate technical efficiency include:

- Radial
- Non-radial
- Hybrid
- Directional Distance Function
- Cost (Type I and type II)
- Revenue(Type I and type II)
- Profit (Type I and type II)

- Revenue/Cost Ratio (Type I and type II)

Radial

It measures the necessary proportional improvements of relevant factors (inputs / outputs) for the evaluated DMU to reach the frontier.

Non-radial

Non-radial model is also called slack based model (SBM). It maximizes the average improvements of relevant factors (inputs / outputs) for the evaluate DMU to reach the frontier.

Hybrid

Hybrid model is a mixture of radial model and Non-radial model, and it is a generalized form of radial and Non-radial models. If Hybrid model is selected, you should define the distances of inputs/outputs. Click the "Define" button right to the Hybrid check box, the distance definition form will open.

If all the inputs/outputs are set to be radial in Hybrid model, it is equivalent to radial model, and if all set to be Non-radial, it is equivalent to SBM model.

Directional Distance Function

Directional distance function is a generalized form of radial model. MaxDEA provides complete support for directional distance function model. It provides a more generalized form of directional distance function defined by Chambers, Chung and Fare (1998). Here, α, β and γ are not necessarily have same values, i.e. the proportion of input decrease is not necessarily equal to that of output increase, and good output increase is not necessary equal to bad output decrease.
The directional distance function model in MaxDEA is expressed as

For DMU_0

$$\text{Efficiency score} = \min \frac{1-\frac{1}{m}\sum_{i=1}^{m} w_i \alpha g_{xio} / x_{io}}{1+o_d \frac{1}{s_d}\sum_{d=1}^{s_d} w_d \beta g_{ydo} / y_{do} + o_u \frac{1}{s_u}\sum_{u=1}^{s_u} w_u \gamma g_{yuo} / y_{uo}}$$

s.t. $X\lambda + \alpha g^x \le x_o$

$Y^d \lambda - \beta g_y^d \ge y_o^d$

$Y^u \lambda + \gamma g_y^u \le y_o^u$

m: the number of inputs;

s_d: the number of desirable (good) outputs

s_u: the number of undesirable (bad) outputs;

w_i: user-defined weight of inputs;

w_d: user-defined weight of desirable (good) outputs;

w_u: user-defined weight of undesirable (bad) outputs;

x_0 and y_0: inputs and outputs of the evaluated DMU_0;

g_x and g_y: user-defined direction vector for inputs and outputs;

o_d: user-defined overall weight of desirable (good) outputs;

o_u: user-defined overall weight of undesirable (bad) outputs

$$\sum_{i=1}^{m} w_i = m, \quad \sum_{d=1}^{s_d} w_d = s_d, \quad \sum_{u=1}^{s_u} w_u = s_u \text{ ;}$$

$o_d + o_u = 1$.

In MaxDEA, the user can set the weights freely, for example, suppose there are 2 inputs and 2 outputs in the model, you can set weight for input1 = 1, weight for input1 = 2, weight for output1 = 1, weight for output2 = 2, o_d =1, o_u=1, and MaxDEA will calculate the proper weights automatically.

This method to compute the efficiency score is defined by the authors of MaxDEA, Gang Cheng and Zhenhua Qian, with the following features:

1) The efficiency scores computed with this method are independent of the length of the direction vector, although the values of α and β are dependent of the length of the direction vector. For example, the computed efficiency scores are the same whether the direction vector (1, 1, 1, 1) or (2, 2, 2, 2) is used.

2) It keeps consistent with the measure used in radial model. If the directional distance function model defined is equivalent to radial model, the efficiency scores from the two types of models are the same.

3) The inputs and outputs can be attached with user-defined weights to indicate their relative importance.

4) Overall weights of good outputs and bad outputs can be defined.

If the user wants to compute the efficiency scores using his own formula, he can do it with the values of α, β and γ provided in the results.

There are five types of direction vectors to use for the directional distance function in MaxDEA.

1) The values of the evaluated DMU;

2) The mean of all DMUs;

3) Vector (1, 1, 1…);

4) A self-defined vector same to all DMUs;

5) Self-defined vectors different to different DMUs.

The fourth type is a generalized form of the second, and the last is a generalized form of all the others.

Note that the first two types are independent of units of inputs and outputs.

The directional distance function model will be equivalent to the radial model with the first type of direction vector, i.e. $g_x = x$, and $g_y = y$.

Reference

Chambers RG, Chung Y, Fare R. Profit, directional distance functions, and Nerlovian efficiency. J Optimiz Theory App. 1998; 98(2): 351-64.

Cost, Revenue, Profit, and Revenue/Cost Ratio Models

These models deal with financing aspects of DEA models in the case that prices of inputs/outputs are known. It is related to the topic of allocative efficiency.

There are two types of LPs for the above models, and MaxDEA names them as "Type I "and "Type II" respectively. The difference between type I and type II is that type I models use the original inputs/outputs values in constraints, while type II models use cost/revenue values of inputs/outputs in constraints. (Type I is traditional and commonly used.)

To run the above models, price information must be set first using the "Define" button on the right side. Cost model needs input prices, Revenue model needs output prices, and Profit and Revenue/Cost Ratio models need both input and output prices. The price fields must be kept "Not define" at the stage of "Define Data".

Note that efficiency score of profit model might be **negative**.

Orientation

Traditional orientations include

1) input-orientation,
2) output-orientation and
3) non-orientation.

MaxDEA provides five new orientations:

4) input-orientation(modified),
5) output-orientation(modified),
6) non-orientation (input-prioritized),
7) non-orientation (output-prioritized),

8) non-orientation (generalized priority).

The significance of the new orientations is that the modified input-oriented and modified output-oriented super-efficiency models overcome the infeasibility problem in the traditional super-efficiency models.

The non-orientation with generalized priority is a generalized form of other orientations.

The radial model with generalized orientation is expressed as

For DMU_k

$$\min \ \frac{1-w^I\alpha}{1+w^O\beta}$$

$$\text{s.t.} \ \sum_{j=1}^{n}\lambda_j x_{ij} \le (1-\alpha)x_{ik}, \ \ i=1,2,...,m \ (if \ w^I>0)$$

$$\sum_{j=1}^{n}\lambda_j x_{ij} \le x_{ik}, \ \ i=1,2,...,m \ (if \ w^I=0)$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} \ge (1+\beta)y_{rk}, \ \ r=1,2,...,s \ (if \ w^O>0)$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{rk}, \ \ r=1,2,...,s \ (if \ w^O=0)$$

$$\sum_{j=1}^{n}\lambda_j = 1$$

$$\lambda_j \ge 0, \ j = 1, 2, \ldots, n$$

where w^I and w^O are user-defined non-negative numbers and at least one of them is positive, and the efficiency score is defined as $(1-\alpha)/(1+\beta)$.

If DMU_k is efficient in the above model, its super-efficiency model is expressed as

$$\min \ \frac{1-w^I\alpha}{1+w^O\beta}$$

$$\text{s.t.} \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j x_{ij} \le (1-\alpha)x_{ik}, \quad i=1,2,...,m \ (if \ w^I>0)$$

$$\sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j x_{ij} \le (1-\alpha)x_{ik}, \quad i=1,2,...,m \ (if \ w^I=0)$$

$$\sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j y_{rj} \ge (1+\beta)y_{rk}, \quad r=1,2,...,s \ (if \ w^O>0)$$

$$\sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j y_{rj} \ge y_{rk}, \quad r=1,2,...,s \ (if \ w^O=0)$$

$$\sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j = 1$$

$$\alpha \le 0, \beta \le 0, \lambda_j \ge 0, \text{ j = 1, 2, . . . , n}(j \neq k)$$

where w^I and w^O are user-defined non-negative numbers and at least one of them is positive, and the super-efficiency score is defined as (1-α)/(1+β).

Super-efficiency model may not have feasible solutions under variable, non-increasing and non-decreasing returns to scale (VRS, NIRS, NDRS) technology. Seiford and Zhu (1999) provide the necessary and sufficient conditions for the infeasibility of super-efficiency.

In the above generalized model, w^I and w^O denotes the priority of orientation. The generalized super-efficiency model is always feasible when both w^I and w^O are positive values.

Table 3-4(A) shows the relationship between the generalized orientation and the other seven orientations

Case 1: Input-orientation. w^I = 1, w^O = 0, and the (super-) efficiency score is defined as $1 - \alpha^*$

In case 1, the generalized model is equivalent to the traditional input-oriented model, and the (super-) efficiency score $1 - \alpha^*$ in the generalized model is equal to θ^* in the traditional input-oriented model.

Case 2: Output-orientation. $w^I = 0$, $w^O = 1$, and the (super-) efficiency score is defined as $1/(1 + \beta^*)$

In case 2, the generalized model is equivalent to the traditional output-oriented model, and the (super-) efficiency score $1/(1 + \beta^*)$ in the generalized model is equal to$1/\varphi^*$ in the traditional output-oriented model.

In addition to input-orientation and output-orientation, we define five more orientations.

Case 3: Non-orientation. $w^I = 1$, $w^O = 1$, and the (super-) efficiency score is defined as $(1 - \alpha^*)/(1 + \beta^*)$

When input-orientation and output-orientation are given equal priority ($w^I = 1$ and $w^O = 1$), the model is non-oriented, and the (super-) efficiency score is defined as $(1 - \alpha^*)/(1 + \beta^*)$.

Case 4: Input-orientation(modified). $w^I = 1$, $w^O = \varepsilon$ (standard efficiency) or $w^I = \varepsilon$ and $w^O = 1$ (super-efficiency), and the (super-) efficiency score is defined as $1 - \alpha^*$

In case 4, input-orientation is given priority with output-orientation retained. The (super-) efficiency score is defined as $(1 - \alpha^*)$ without the denominator $(1 + \beta^*)$, which means that the efficiency score is measured by movements of inputs only. It is a modified input-orientation with the following properties: the standard efficiency score $(1 - \alpha^*)$ is equal to θ^* in the traditional input-oriented model, and

the super-efficiency score is equal to θ^* in the traditional input-oriented model as well when it is feasible. In addition, when the traditional input-oriented super-efficiency model is infeasible, this modified input-oriented model will still yield an optimal solution. The modified input-oriented model overcomes the problem of infeasibility, and at the same time, it keeps the concordance between the modified model and the traditional model.

Case 5: Output-orientation(modified). $w^I = \varepsilon$, $w^O = 1$ (standard efficiency) or $w^I = 1$ and $w^O = \varepsilon$ (super-efficiency), and the (super-) efficiency score is defined as $1/(1 + \beta^*)$.

In case 5, output-orientation is given priority with input-orientation retained. The (super-) efficiency score is defined as $1/(1 + \beta^*)$ without the numerator $(1 - \alpha^*)$, which means that the efficiency score is measured by movements of outputs only. It is a modified output-orientation with the following properties: the standard efficiency score$1/(1 + \beta^*)$ is equal to $1/\varphi^*$ in the traditional output-oriented model, and the super-efficiency score is equal to $1/\varphi^*$ in the traditional output-oriented model as well when it is feasible. In addition, when the traditional output-oriented super-efficiency model is infeasible, this modified output-oriented model will still yield an optimal solution. The modified output-oriented model overcomes the problem of infeasibility, and at the same time, it keeps the concordance between the modified model and the traditional model.

Case 6: Non-orientation (input-prioritized). $w^I = 1$, $w^O = \varepsilon$ (standard efficiency) or $w^I = \varepsilon$ and $w^O = 1$ (super-efficiency), and the (super-) efficiency score is defined as $(1 - \alpha^*)/(1 + \beta^*)$

Case 6 is identical to case 4 except that the efficiency score is defined as $(1 - \alpha^*)/(1 + \beta^*)$, which means that the efficiency score is measured by movements of both inputs and outputs. It is an input-prioritized non-orientation with the

following properties: the standard efficiency score $(1 - \alpha^*)/(1 + \beta^*)$ is equal to or less than θ^* in the traditional input-oriented model, and the super-efficiency score $(1 - \alpha^*)/(1 + \beta^*)$ is equal to or greater than θ^* in the traditional input-oriented model when it is feasible.

Case 7: Non-orientation (output-prioritized). $w^I = \varepsilon$, $w^O = 1$ (standard efficiency) or $w^I = 1$ and $w^O = \varepsilon$ (super-efficiency), and the (super-) efficiency score is defined as $(1 - \alpha^*)/(1 + \beta^*)$

Case 7 is identical to case 5 except that the efficiency score is defined as $(1 - \alpha^*)/(1 + \beta^*)$, which means that the efficiency score is measured by movements of both outputs and outputs. It is an output-prioritized non-orientation with the following properties: the standard efficiency score $(1 - \alpha^*)/(1 + \beta^*)$ is equal to or less than $1/\varphi^*$ in the traditional output-oriented model, and the super-efficiency score $(1 - \alpha^*)/(1 + \beta^*)$ is equal to or greater than $1/\varphi^*$ in the traditional output-oriented model when it is feasible.

Table 3-4(A) Special cases of the generalized model and their definitions for efficiency score

(the relationship between the generalized orientation and the other seven orientations)

Case	Orientation	Standard efficiency model			Super-efficiency model		
		w^I	w^O	Score	w^I	w^O	Score
1	Input-oriented	1	0	$1-\alpha^*$	1	0	$1-\alpha^*$
2	Output-oriented	0	1	$\frac{1}{1+\beta^*}$	0	1	$\frac{1}{1+\beta^*}$
3	Non-oriented	1	1	$\frac{1-\alpha^*}{1+\beta^*}$	1	1	$\frac{1-\alpha^*}{1+\beta^*}$

4	Input-oriented (modified)	1	ε	$1-\alpha^*$	ε	1	$1-\alpha^*$
6	Non-oriented (input-prioritized)	1	ε	$\frac{1-\alpha^*}{1+\beta^*}$	ε	1	$\frac{1-\alpha^*}{1+\beta^*}$
5	Output-oriented (modified)	ε	1	$\frac{1}{1+\beta^*}$	1	ε	$\frac{1}{1+\beta^*}$
7	Non-oriented (output-prioritized)	ε	1	$\frac{1-\alpha^*}{1+\beta^*}$	1	ε	$\frac{1-\alpha^*}{1+\beta^*}$

ε is the non-Archimedean infinitesimal (in practice, we use 10^-5).

Similarly, the Non-radial model with generalized orientation is expressed as

$$\min \quad \rho = \frac{1 - w^I \left(\frac{1}{m}\sum_{i=1}^{m} s_i^- / x_{io}\right)}{1 + w^O \left(\frac{1}{s}\sum_{r=1}^{s} s_r^+ / y_{ro}\right)}$$

$$st \quad x_0 - X\lambda - s^- = 0$$

$$Y\lambda - y_0 - s^+ = 0$$

$$\lambda, s^-, s^+ \geq 0$$

The equations of the DEA model with generalized orientation is nonlinear programming. It can be transformed into the linear programming using a method similar to Charnes–Cooper transformation (Charnes & Cooper, 1962).

Note: The above methods for the new orientations are developed by Dr Cheng Gang and Dr Qian Zhenhua, the authors of MaxDEA. The details on the methods will be published in international journals. If needed, please cite this method as: *Cheng G, Qian Z, Guo Y (2010). Overcoming the infeasibility of super-efficiency DEA model: a model with generalized orientation. Proceedings of the 8th*

international conference on data envelopment analysis.

Returns to scale (RTS)

Five types of RTS (returns to scale) are available with MaxDEA: constant(CRS), variable(VRS), nonincreasing(NIRS), nondecreasing(NDRS) and generalized(GRS). If GRS is selected, you should also set the lower bound (L) and upper bound (U) of Σλ. GRS is a generalized form of the other four types of RTS, with the following relationship:

Table 3-4(B) Relationship between GRS and other RTS

	GRS	
	L	**U**
CRS	0	+∞
VRS	1	1
NIRS	0	1
NDRS	1	+∞

The option "Scale Efficiency" is a special case in which both CRS and VRS models will be solved, and the scale efficiency or scale effect in Malmquist model will be computed.

Scale Efficiency = CRS efficiency / VRS efficiency

RTS Estimation is also provided with this option.

For Malmquist model, the relationship between scale factors are as follows

- Scale Effect on Malmquist Index = Malmquist Index (CRS) / Malmquist Index (VRS)
- Scale Effect on Efficiency Change = Efficiency Change (CRS) / Efficiency Change (VRS)
- Scale Effect on Technological Change = Technological Change (CRS) / Technological Change (VRS)

- Scale Effect on Malmquist Index = Scale Effect on Efficiency Change * Scale Effect on Technological Change

See also 3.3.1.1.2 Malmquist Model and in 3.3.1.2.2 Scale Efficiency

FDH Model

The purpose of Free Disposal Hull (FDH) model is to ensure that efficiency evaluation is based on only actually observed performances. The input-oriented FDH model can be expressed using the following mixed integer LP,

$$\min \ \theta$$

$$st \ \ \theta x_0 - X\lambda - s^- = 0$$

$$Y\lambda - y_0 - s^+ = 0$$

$$s^-, s^+ \geq 0$$

$$e\lambda = 1, \ \lambda \in \{0,1\}$$

Note: FDH model is available only under the following basic specifications:

Returns To Scale: VRS

Orientation: input-oriented or output-oriented.

3.3.1.1.2 Advanced Specifications(Advanced Model)

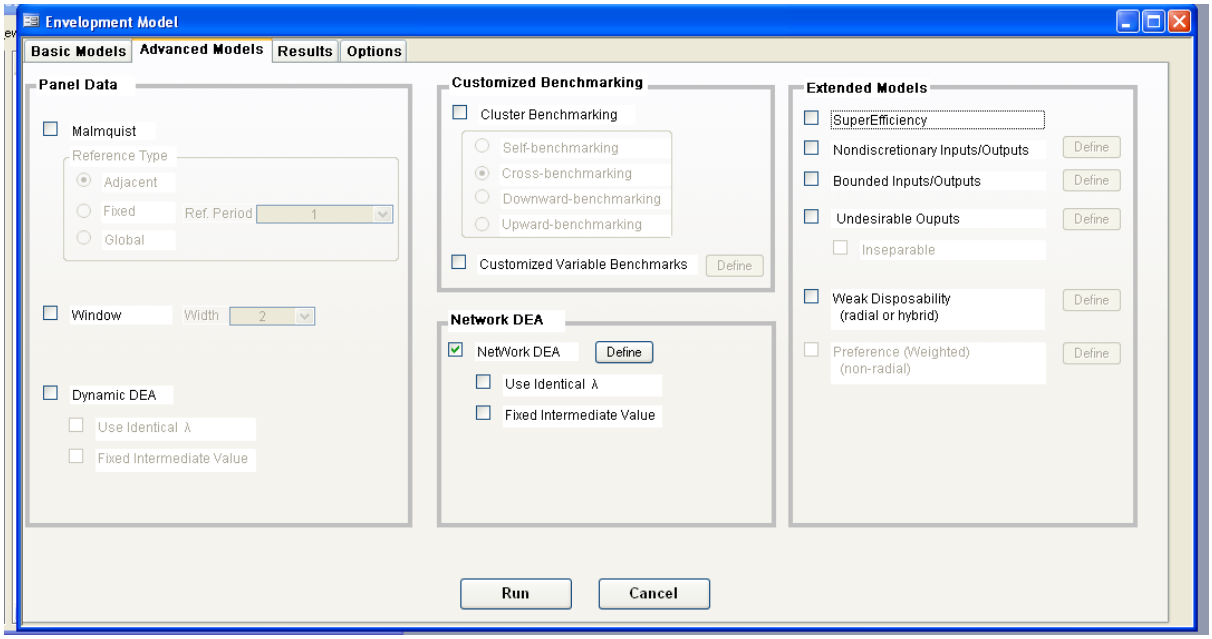


Figure 3-1-B Advanced specifications for Envelopment model

Super-efficiency

The difference between Super-efficiency model and standard efficiency model is that in super models the DMU_0 (the DMU evaluated) is eliminated from the reference set (indicated by $j \neq 0$ in the LP).

The Super-efficiency score can be greater than 1. In some cases, the LP for some DMUs will be infeasible. In such cases, you can decide whether the program returns 1 as the score through the option "No optima" (see 3.3.1.3).

Malmquist Model

The Malmquist model deals with panel data. The Malmquist productivity index evaluates the total factor productivity change of a DMU between two periods. It is defined as the product of efficiency change (catch-up) and technological change (frontier-shift). The efficiency change reflects to what extent a DMU improves or worsens its efficiency, while technological change reflects the change of the efficiency frontiers between two periods.

MaxDEA provides three types of Malmquist Indices with three kinds of reference: adjacent, fixed and global.

(See details in 3.3.1.2.2 RESULTS FOR MALMQUIST MODELS)

References for these three types of Malmquist

Adjacent

FGLR: Färe R, Grosskopf S, Lindgren B, Roos P, 1992

Fixed

Berg SA, Forsund FR, Jansen ES, 1992

Global

Pastor JT, Lovell CAK, 2005

At present, the Adjacent Malmquist is the commonly used type.

MaxDEA provides Malmquist Index and its components (efficiency change and technological change) for not only CRS but also VRS and other types of RTS.

MaxDEA provides three indicators for the scale effect in Malmquist model, with the following relationships

- SEEC:

  Scale Effect on Efficiency Change = Efficiency Change (CRS) / Efficiency Change (VRS)

- SETC:

  Scale Effect on Technological Change = Technological Change (CRS) / Technological Change (VRS)

- SEMI:

  Scale Effect on Malmquist Index = Malmquist Index (CRS) / Malmquist Index (VRS)

- Scale Effect on Malmquist Index = Scale Effect on Efficiency Change *

Scale Effect on Technological Change

Note that SEEC is equivalent to “Scale Change Factor (SCH)” defined by FGNZ, and SEMI is equivalent to “Scale Change Factor (SCH)” defined by Ray and Desli.

(See also 3.3.1.2.2 RESULTS FOR MALMQUIST MODELS)

References

FGLR:

Färe R, Grosskopf S, Lindgren B, Roos P. Productivity changes in Swedish pharamacies 1980–1989: A non-parametric Malmquist approach. J Prod Anal. 1992; 3(1-2): 85-101.

FGNZ:

Fare R, Grosskopf S, Norris M, Zhang Z. Productivity Growth, Technical Progress, and Efficiency Change in Industrialized Countries. Am Econ Rev. 1994; 84(1): 66-83.

Ray and Desli:

Ray SC, Desli E. Productivity growth, technical progress, and efficiency change in industrialized countries: Comment. Am Econ Rev. 1997;87(5):1033-9.

Berg SA, Forsund FR, Jansen ES. Malmquist Indexes of Productivity Growth during the Deregulation of Norwegian Banking, 1980-89. Scand J Econ. 1992; 94: S211-S28

Pastor JT, Lovell CAK. A global Malmquist productivity index. Economics Letters. 2005; 88(2): 266-71.

Cost Malmquist Model

MaxDEA uses the method developed by Maniadakis and Thanassoulis to compute Cost Malmquist Index, and uses similar methods to compute Revenue, Profit, and Revenue Cost Ratio Malmquist Indices.

MaxDEA directly provides the first stage decomposition of Cost Malmquist

Index as follows

- Malmquist Index(cost efficiency model) = Efficiency Change (cost efficiency model) * Technological Change (cost efficiency model)

The second stage decomposition can be attained by running both cost Malmquist Model and technical efficiency Malmquist Model, and computing its components as follows

- Allocative Efficiency Change = Efficiency Change (cost efficiency model) / Efficiency Change (technical efficiency model)
- Price Effect = Technological Change (cost efficiency model) / Technological Change (technical efficiency model)
- Malmquist Index(cost efficiency model) = Efficiency Change (technical efficiency model) * Allocative Efficiency Change * Technological Change (noraml) * Price Effect

Revenue, Profit and Revenue Cost Ratio Malmquist Indices can be decomposed similarly.

Maniadakis N, Thanassoulis E. A cost Malmquist productivity index. Eur J Oper Res. 2004; 154(2): 396-409.

Revenue Malmquist Model

Similar to Cost Malmquist Model.

Profit Malmquist Model

Similar to Cost Malmquist Model.

Revenue Cost Ratio Malmquist Model

Similar to Cost Malmquist Model.

Window Model

The window model also deals with panel data. Window width must be set for window models. The panel data may be unbalanced for window analysis, i.e. the

number of DMUs in each period may be different, which is not permitted for Malmquist model..

Non-controllable Inputs/outputs Model

See “Nondiscretionary Inputs/outputs Model”

Measure Specific Model

See “Nondiscretionary Inputs/outputs Model”

Nondiscretionary Inputs/outputs Model

MaxDEA provides a generalized nondiscretionary inputs/outputs models. And the nondiscretionary model is actually a special case of a more generalized model - bounded model (see the next part), so we just provide the LPs for bounded models

To run the nondiscretionary model, you should first set the discretion status of inputs/outputs (“Full Discretion” means complete control, and “Non- or Part-discretion” means limited control), and set the discretion degree for nondiscretionary inputs/outputs. To do so, just click the “Define” button on the right side. The format of discretion degree is percent.

Let’s talk about some special cases of nondiscretionary models.

1) The first special case is the nondiscretionary radial model with the discretion degrees of all nondiscretionary inputs/outputs being zero. Such a case is also called “**non-controllable**” radial model (Cooper William W，Seiford Lawrence M，Tone Kaoru, 2007).
2) The second special case is the nondiscretionary radial model with the discretion degrees of all nondiscretionary inputs/outputs being 100%. Such a case is also called “**non-discretionary**” radial model (Cooper William W，Seiford Lawrence

M，Tone Kaoru, 2007), or "**measure specific**" model (Zhu J, 2009).

Please note that setting the discretion degrees of nondiscretionary inputs/outputs to be 100% is **not** equivalent to setting the inputs/outputs to be "full-discretion" in radial models, which is a little puzzling.

3) The third special case is the nondiscretionary SBM model with the discretion degrees of all nondiscretionary inputs/outputs being zero. Such a case is also called "**non-controllable**" SBM model(Cooper William W，Seiford Lawrence M，Tone Kaoru, 2007).
4) The last special case is the nondiscretionary SBM model with the discretion degrees of all nondiscretionary inputs/outputs being 100%. Such a case is equivalent to a normal model. In other words, setting the discretion degrees of nondiscretionary inputs/outputs to be 100% is equivalent to setting the inputs/outputs to be "Full Discretion" in SBM models.

If all the inputs/outputs are set to be "Full Discretion", it is equivalent to the normal model.

Bounded Inputs/outputs Model

To run a bounded model, you should first set the lower and upper bounds of the bounded inputs/outputs, by the "define" button on the right side. The fields indicating lower and upper bounds of inputs/outputs must be kept "Not define" at the stage of "Define Data". Lower bound must be less than or equal to the original value, and upper bound must be greater than or equal to the original value.

If all the inputs/outputs are set to be "Full Discretion", it is equivalent to the normal model.

The nondiscretionary model is equivalent to the bounded model by setting the lower and upper bounds as follows,

Lower bound = original value ×(1－discretion degree)

Upper bound = original value /(1－discretion degree)

Undesirable Outputs Model

Undesirable model deals with the circumstances that bad outputs exit, by setting the improvements of bad outputs in an opposite direction to the good outputs, which means that more good outputs and less bad outputs are desired.

With MaxDEA, you can develop radial, Non-radial or hybrid undesirable models. The radial undesirable model is based on the **directional distance function** (Chung YH, Fare R and Grosskopf S, 1997), and uses a generalized equation, as follows:

The undesirable radial model (nonoriented):

$$\min \quad \rho = \frac{1-\alpha}{1+\dfrac{w_g\beta + w_b\gamma}{w_g + w_b}}$$

$$st \quad (1-\alpha)x_0 - X\lambda - s^- = 0$$

$$Y^g\lambda - (1+\beta)y_0^g - s^{g+} = 0$$

$$Y^b\lambda - (1-\gamma)y_0^b + s^{b+} = 0$$

g indicates good outputs, b indicates bad outputs, w_g indicates weight of good outputs, and w_b indicates weight of bad outputs, and.

If we add the following constraints

1) **α = β**;
2) **β = γ**,
3) $s^{b+} = 0$ (slacks of bad outputs to being zero, i.e. weak disposability, combined with **weak disposability model**), and
4) orientation = non-oriented,

the model will be the traditional "directional distance function" model, as follows:

$$\max \ \beta$$

$$st \ \ X\lambda + s^{-} = (1-\beta)x_0$$

$$Y^g\lambda - s^{g+} = (1+\beta)y_0^g$$

$$Y^b\lambda = (1-\beta)y_0^b$$

$$Efficiency\ Score = \frac{1-\beta}{1+\beta}$$

The undesirable Non-radial(SBM) model (nonoriented):

$$\min \ \ \rho = \frac{1-\frac{1}{m}\sum_{i=1}^{m} s_i^{-} / x_{io}}{1+\frac{1}{s}\sum_{r=1}^{s} s_r^{+} / y_{ro}}$$

$$st \ \ x_0 - X\lambda - s^{-} = 0$$

$$Y^g\lambda - y_0^g - s^{g+} = 0$$

$$Y^b\lambda - y_0^b + s^{b+} = 0$$

$$\lambda, s^{-}, s^{+} \geq 0$$

Click the button "Define" on the right side to set bad outputs, and additional constraints including "α = β", "β = γ" and weak disposability.

Inseparable Good and Bad Outputs Model

Inseparable outputs model deals with the situation that certain bad outputs are inseparable from the corresponding good outputs (and certain inputs). Reducing bad outputs is inevitably accompanied by reduction in good outputs.

Refer to the book by Cooper et al (2007) for details about this model. There

are two types of inseparable models in this book, one is names as “SBM-NS”, and the other is named as “NS-Overall”.

Please note that MaxDEA provides a more flexible inseparable model, you should make the options in MaxDEA according to the linear programs in the literature. The relevant options include **orientation, weak disposability, discretion, and whether**

- ✓ Total amount of good outputs remains unchanged;
- ✓ Slacks of inseparable inputs are treated as inefficiency; and
- ✓ Slacks of inseparable bad outputs are treated as inefficiency.

Note that in MaxDEA the increasing upper bound for outputs is calculated with the following formula

Upper bound = original value /(1－discretion degree).

For example, if the discretion degree is set to be 20%, the actual percentage of increasing will be 1 / (1 – 20%) – 100% = 25%. If you want to set the upper bound of percentage of increasing to be 20%, the discretion degree should be 1 – 1 / (1 + 20%) = 16.6667%.

Weak Disposability Model

The weak disposability model restricts the slacks of input/outputs with weak disposability to being zero, as follows (nonoriented):

$$\min \ \frac{\theta}{\phi}$$

$$st \ \ \theta x_0^s - X^s\lambda - s^{s-} = 0$$

$$\theta x_0^w - X^w\lambda = 0$$

$$Y^s\lambda - \phi y_0^s - s^{s+} = 0$$

$$Y^w\lambda - \phi y_0^w = 0$$

s indicates strong disposability and w indicates weak disposability.

Click the button “Define” on the right side to set inputs/outputs with weak disposability.

Note: Weak Disposability model is NOT available under Non-radial (SBM) specification...

Preference (weighted) Model

Weights can be assigned to inputs and outputs in SBM models according to their relative importance as follows:

$$\min \quad \rho = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} w_i s_i^- / x_{io}}{1 + \frac{1}{s}\sum_{r=1}^{s} w_r s_r^+ / y_{ro}}$$

$$\sum_{i=1}^{m} w_i = m$$

$$\sum_{r=1}^{s} w_r = s$$

Click the button “Define” on the right side to set weights.

The user can set the weights freely, for example, suppose there are 2 inputs and 2 outputs in the model, you can set

weight for input1 = 1,

weight for input1 = 2,

weight for output1 = 1,

weight for output2 = 2,

and MaxDEA will calculate the corresponding w_i and w_r automatically.

If you set the weights of all inputs/outputs to be 1, it is equivalent to the normal model.

Note: Preference model is available only under Non-radial (SBM) specification.

Cost, Revenue, Profit, and Revenue/Cost Ratio Model

See 3.3.1.1.1 Distance

Variable-benchmark Model

See “Customized Benchmarking”

Customized Benchmarking

In customized benchmarking model, you can customize the reference DMU set for calculating efficiency scores. Take the input-oriented CCR model as an example,

$$\min \ \theta$$

$$st \ \ \theta x_0 - X^r \lambda - s^- = 0$$

$$Y^r \lambda - y_0 - s^+ = 0$$

$$\lambda, s^-, s^+ \geq 0$$

The superscript r indicates the DMUs included in the customized reference DMU set.

The above model is also called **variable-benchmark model** (Zhu J, 2009), because any of the DMUs in R might be the benchmark for a DMU evaluation, but it is not necessary for all the DMUs in R to be the benchmarks.

To set a variable-benchmark model, open the menu “Run Envelopment Model”, check the option “Variable-benchmark”, and click the “Define” button to select variable benchmarks.

Network DEA Model

Network models can be combined with all the other envelopment models, such as

Network Malmquist models, Network Cost/Revenue/Profit/ Revenue Cost Ratio models, Network DEA with undesirable outputs, Network DEA with nondiscretionary inputs/outputs.

The Network DEA model deals with such circumstances in which many divisions (they are called as nodes in MaxDEA) are linked with each other, as the following chart shows (Figure 3-2):

1) Each node **may and may not** have its own inputs and outputs, which are called as direct inputs/outputs in Network DEA models;
2) At least one node is linked with another node through intermediate inputs/outputs, which are outputs for one node, and meanwhile are outputs for the other node.

Network DEA models provides the overall efficiency and the efficiencies of the nodes in a systematic framework.

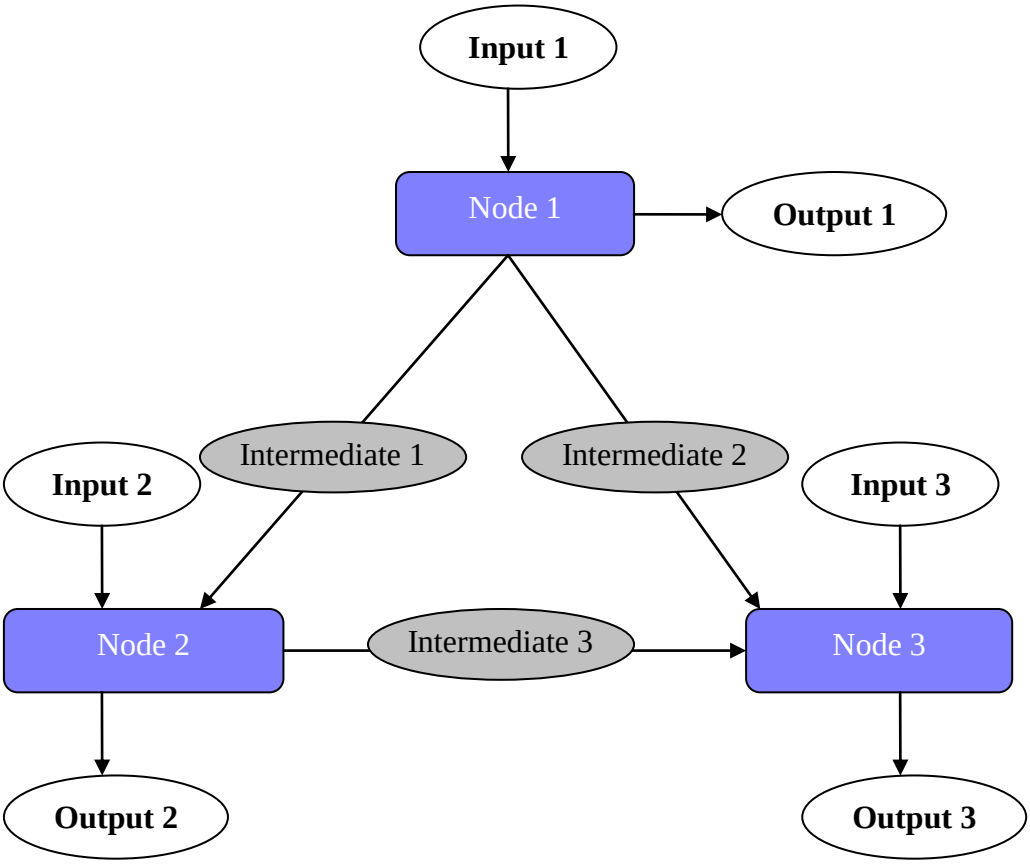


Figure 3-2(A) Sample of Network DEA Model (net-shape)

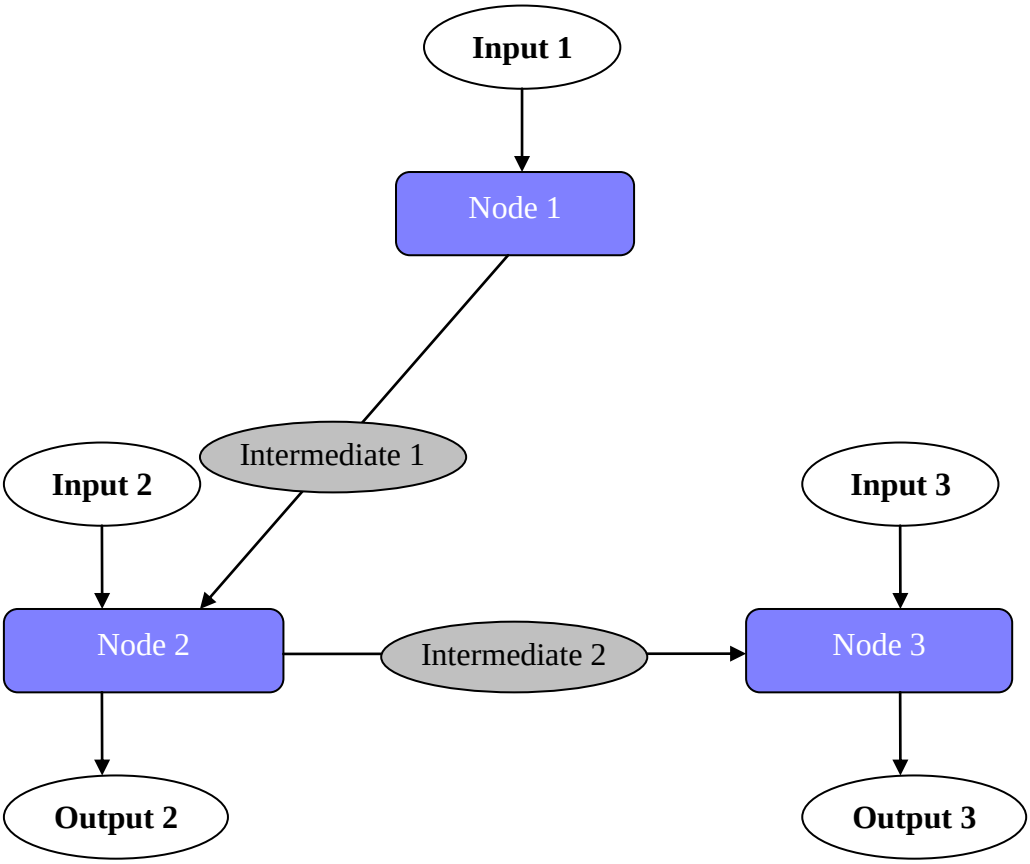


Figure 3-2(B) Sample of Network DEA Model (chain-shape)

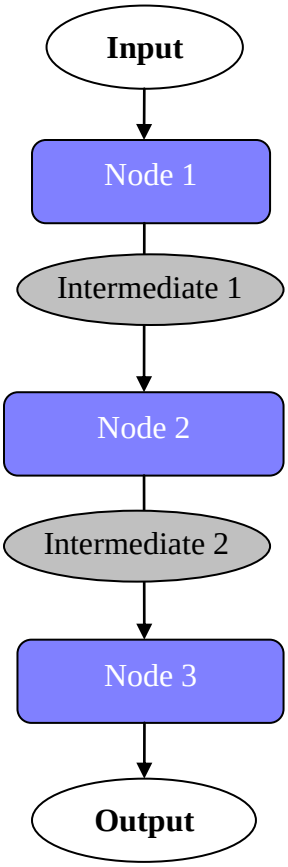


Figure 3-2(C) Sample of Network DEA Model (chain-shape)

MaxDEA is a powerful tool for Network DEA. It provides the most extensive options for Network DEA models.

To run a Network DEA model, you should

1) At the stage of "Define Data", define the direct inputs/outputs just as usual, but define indirect inputs/outputs (intermediate inputs/outputs) as "Intermediate";
2) At the stage of "Run Model", check the box "Network", and
   a) Decide whether different nodes use same or different intensity vector λ through the check box "Use Identical Lambda";
   b) Decide whether the intermediate value is free or fixed through the check box "Fixed Intermediate Value";
   c) Define the Sub-processes of the Network through the "Define" button (Figure 3-4);
      i. Firstly define the nodes for the network, including node name and node weight;
      ii. Secondly, assign the direct inputs, outputs and intermediate to corresponding nodes. Each input or output must be assigned to **one and only one** node, and each intermediate must be assigned to one node as input and another node as output.

d) Run the model.

With MaxDEA, it is very convenient to compare the results between the traditional model (so-called "Black box" model) and the Network model. Just uncheck "Network" with other options unchanged, and it will be the traditional model.

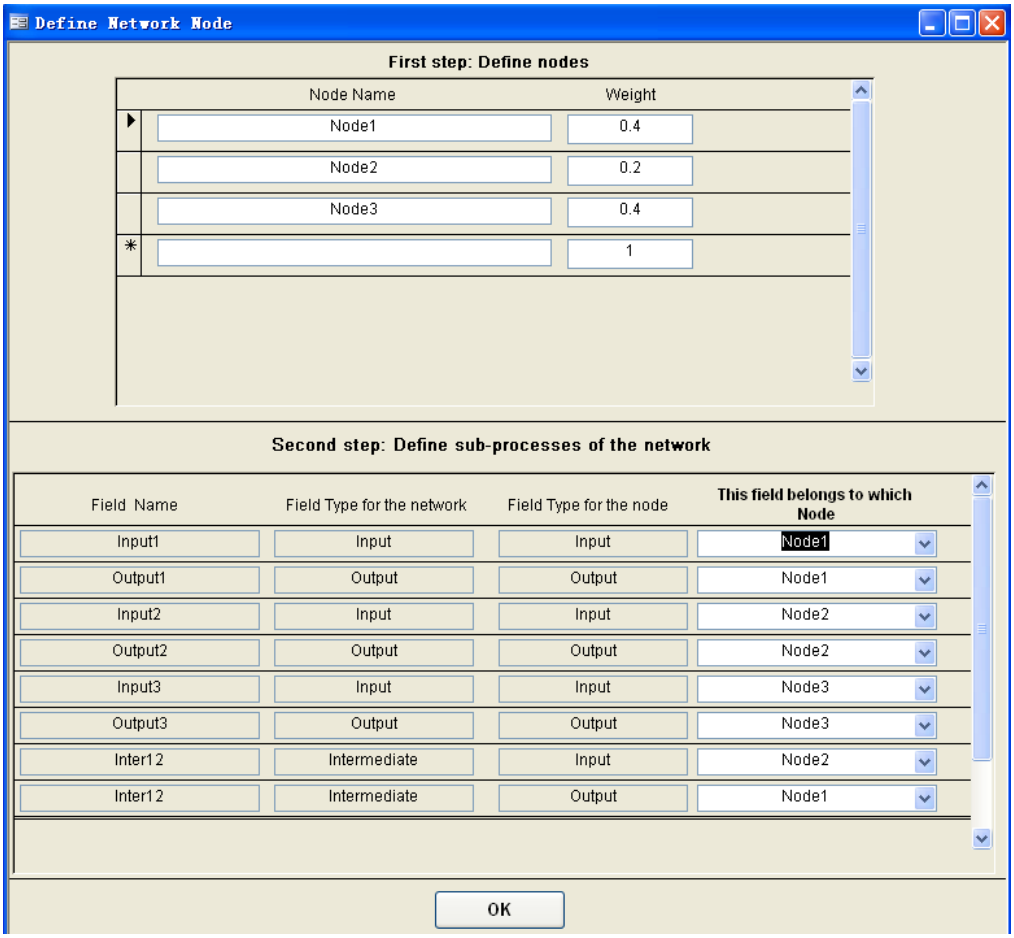


Figure 3-3 Define nodes and sub-processes for Network DEA Model

There are three tables for the results of the Network models: the first is a summary just as usual, the second is overall efficiency and slacks and projections, and the last is efficiency scores of the nodes and the intensity vectors.

Note: Refer to Tone K (2009) for a discussion of Non-radial(SBM) Network DEA.
Tone K, Tsutsui M. Network DEA: A slacks-based measure approach. Eur J Oper Res. 2009; 197(1): 243-52.

Dynamic Model

Dynamic model deals with panel data, and take into consideration the links between periods.
Table 3-5(A) is a panel dataset with one input, one output and one intermediate with

three periods for dynamic model. Figure 3-4 shows the links between periods. Note that in MaxDEA, the intermediate in period 1 is treated as the link between period 1 and 2, the intermediate in period 2 as the link between period 2 and 3, and the intermediate in period 3 is not included in the dynamic model.

Essentially, dynamic model is a special case of network model, so all dynamic models can be equivalently achieved through network model. To do so, the panel data (long data) must be transformed into wide data for network model. Table 3-5(B) is the equivalent dataset to table 3-5-(A) for the same dynamic model achieved though network model, which treats the periods as the nodes.

With MaxDEA, more complex dynamic model , such as dynamic model with different intermediates, inputs and outputs in each period, can be developed through network model.

Table 3-5(A) Panel data for dynamic model (example)

Period	DMU	Input	Output	Intermediate
1	A	4323	93608	875
1	B	2295	225559	469
1	C	6379	327068	1286
2	A	6644	201354	1339
2	B	1436	188926	297
2	C	6281	413738	1266
3	A	7459	114022	1502
3	B	4464	212444	903
3	C	4524	462677	915

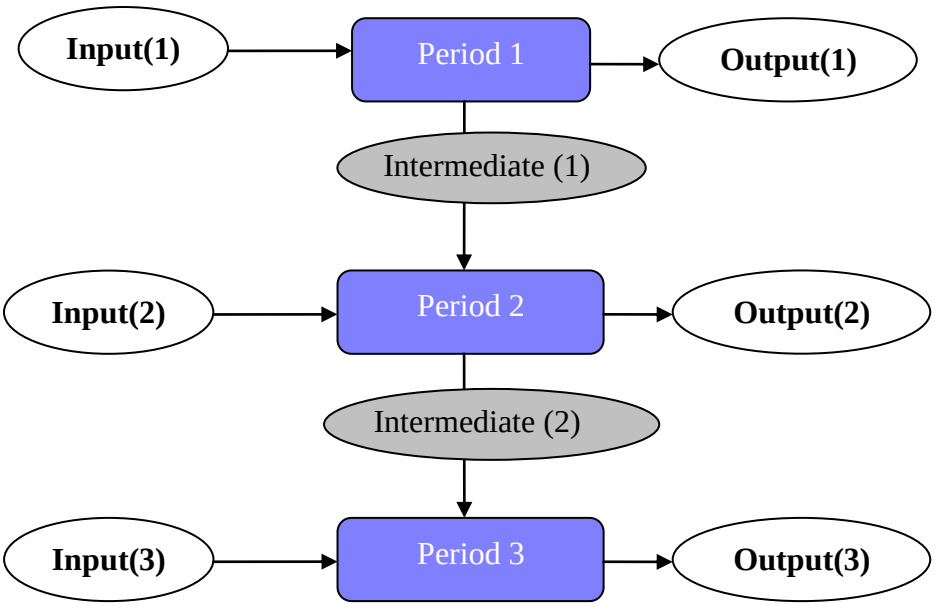


Figure 3-4 Structure of the dynamic model

Table 3-5(B) The equivalent dataset to table 3-5-(A) for the same dynamic model achieved though network model

DMU	Input(1)	Output(1)	Intermediate(1)	Input(2)	Output(2)	Intermediate(2)	Input(3)	Output(3)
A	4323	93608	875	6644	201354	1339	7459	114022
B	2295	225559	469	1436	188926	297	4464	212444
C	6379	327068	1286	6281	413738	1266	4524	462677

Note: Refer to Tone K and Tsutsui M (2010) for a discussion of Non-radial dynamic DEA models.
Tone K, Tsutsui M. Dynamic DEA: A slacks-based measure approach. Omega-Int J Manage Sci. 2010; 38(3-4): 145-56.

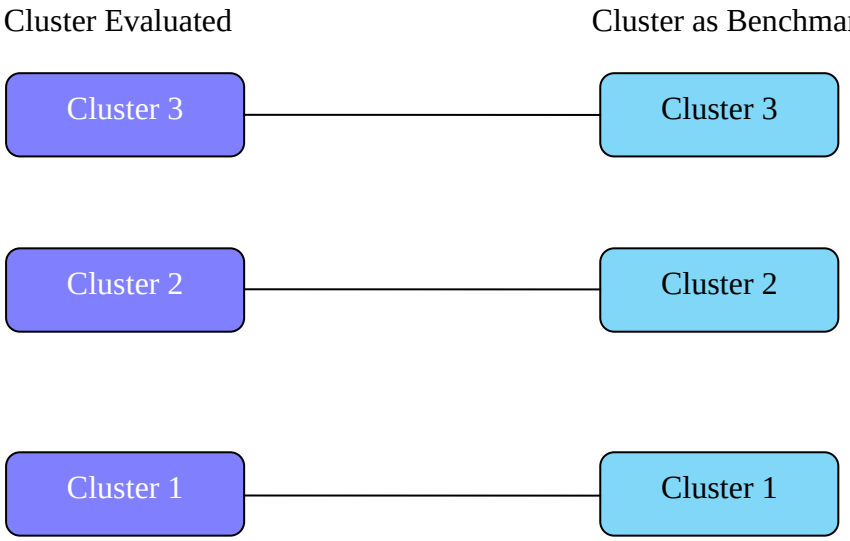
Cluster Model

Cluster Model deals with the situation that the DMUs are categorized according to their characteristics. There are 4 types of cluster models according to the relationship between the clusters evaluated and the clusters as benchmarks:

Self-benchmarking

In self-benchmarking model, each DMU is evaluated with the cluster it belongs to as the reference set. In effect it is a batch mode of evaluating the DMUs by cluster (one by one) using the routine method.
For example, if all the DMUs are categorized into 3 clusters, the results of self-benchmarking model are the same as the routine modes which firstly run the model using the data of cluster 1 only, secondly cluster 2 only, and lastly cluster 3 only.

Self-benchmarking



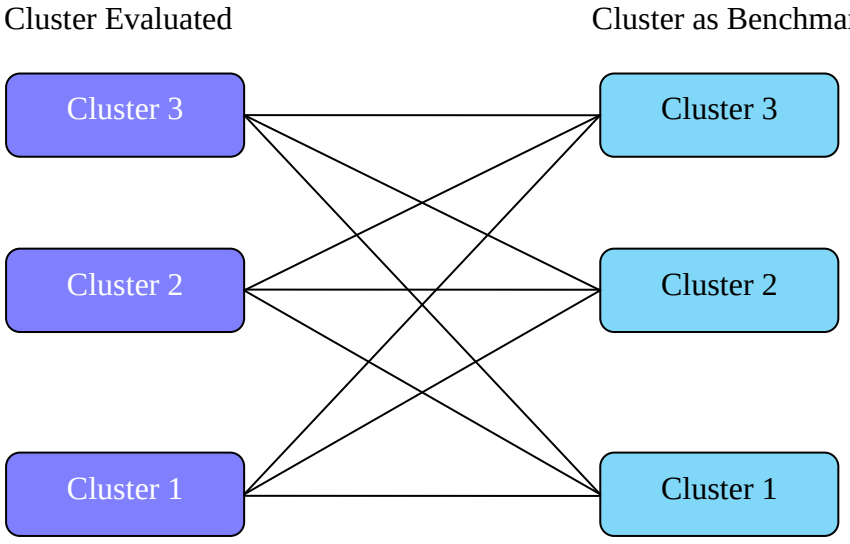
Cross-benchmarking

In cross-benchmarking model, each DMU is evaluated successively with every cluster as the reference set separately.

For example, if all the DMUs are categorized into 3 clusters, the results of cross-benchmarking model are the same as the routine modes as follows,

1) Firstly run the model using the DMUs in cluster 1 as reference set,
2) Secondly using the DMUs in cluster 2 as reference set, and
3) Lastly using the DMUs in cluster 3 as reference set.

Cross-benchmarking



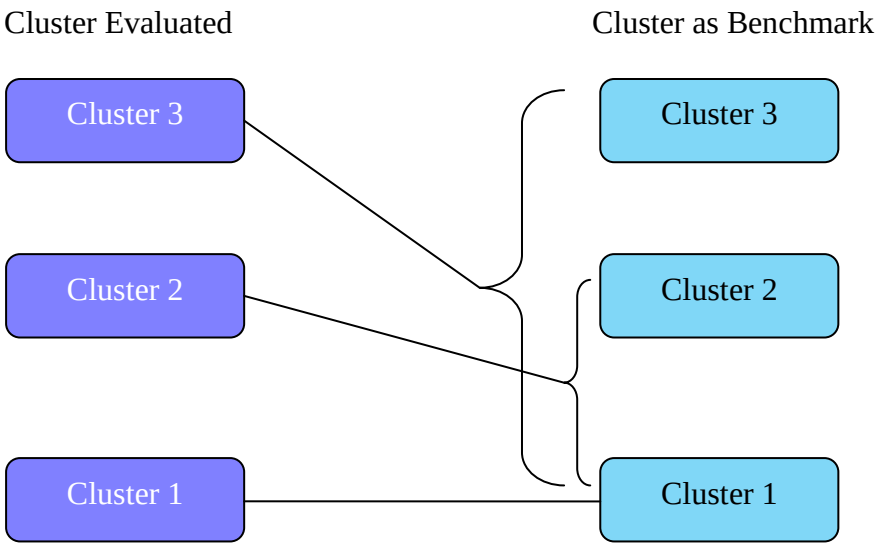
Downward-benchmarking

In downward-benchmarking, the DMUs in a cluster are evaluated with the DMUs in its own cluster and those in the cluster with a lower cluster number (cluster ID) as the reference set.

For example, if all the DMUs are categorized into 3 clusters: clusters 1, clusters 2 and clusters 3,

1) the DMUs in cluster 1 are evaluated with the DMUs in cluster 1 only as the reference set;
2) the DMUs in cluster 2 are evaluated with the DMUs in cluster 1 and cluster 2 as the reference set; and
3) the DMUs in cluster 3 are evaluated with the DMUs in cluster 1, cluster 2, and cluster 3 (all the DMUs) as the reference set.

Downward -benchmarking



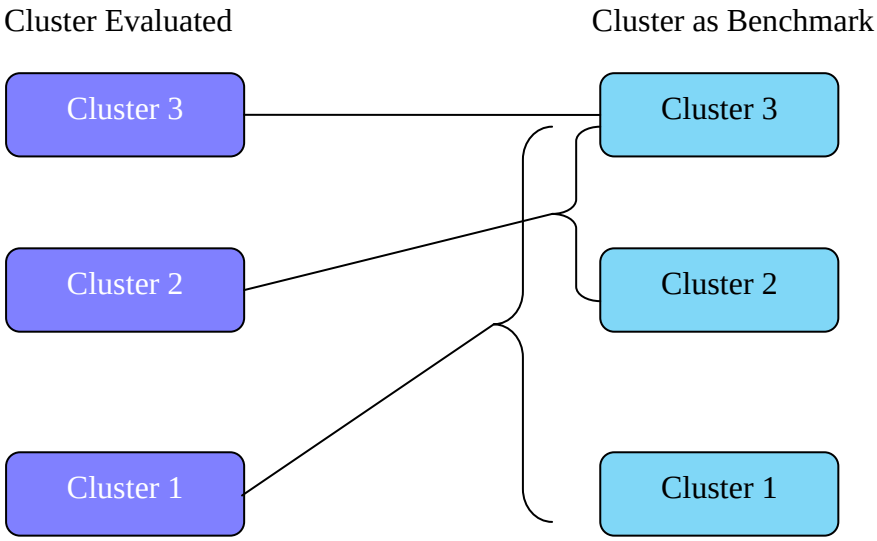
Upward-benchmarking

Contrary to downward-benchmarking, in upward-benchmarking, the DMUs in a cluster are evaluated with the DMUs in its own cluster and those in the cluster with a upper cluster number (cluster ID) as the reference set.

For example, if all the DMUs are categorized into 3 clusters: clusters 1, clusters 2 and clusters 3,

1) the DMUs in cluster 1 are evaluated with the DMUs in cluster 1, cluster 2, and cluster 3 (all the DMUs) as the reference set;
2) the DMUs in cluster 2 are evaluated with the DMUs in cluster 2 and cluster 3 as the reference set; and
3) the DMUs in cluster 3 are evaluated with the DMUs in cluster 3 only as the reference set.

Upward-benchmarking



Note: in the stage of “Define Data”, the field indicating cluster ID should be defined as “Cluster”.

3.3.1.2 Result Specifications

With MaxDEA, it is convenient to customize your results to be shown. The Results tab is for result specifications.

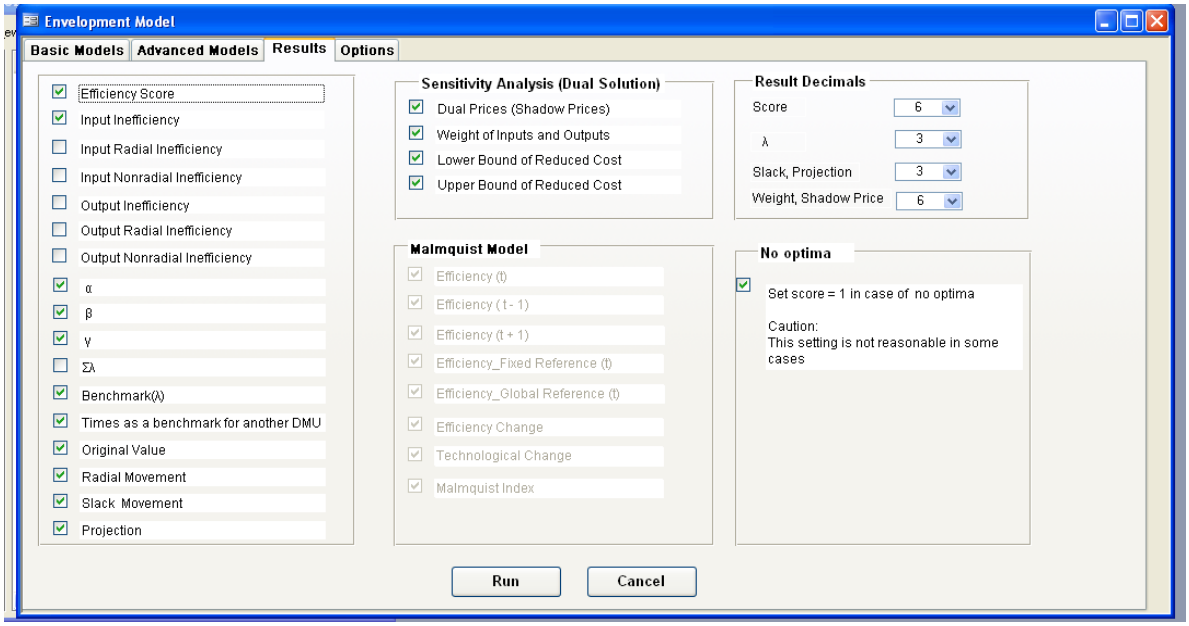


Figure 3-5 Results for Envelopment model

3.3.1.2.1 General Results

Efficiency Score

Please note that MaxDEA uses θ as efficiency score in input-oriented model, and uses 1/φ (the reciprocal of φ) as efficiency score in output-oriented model.

For non-oriented models,

$$\text{Efficiency Score} = \frac{1 - \text{Input Inefficiency}}{1 + \text{Output Inefficiency}}$$

Scale Efficiency

Scale Efficiency = CRS efficiency / VRS efficiency

See also 3.3.1.1.1 RTS

Allocative Efficiency

Allocative Efficiency is not provide directly in MaxDEA. It can be calculated as follows

Allocative Efficiency = Cost (or Revenue or Profit) Efficiency / Technical Efficiency

Input Inefficiency

= Input Radial Inefficiency + Input Non-radial Inefficiency

Input Radial Inefficiency (for Hybrid model)

$$= \frac{m_1}{m}(1-\theta)$$

m: number of inputs; m_1: number of radial inputs

Input Non-radial Inefficiency(for Hybrid model)

$$= \frac{1}{m}\sum_{i=1}^{m_2} s_i^{NR-} / x_{io}^{NR}$$

m: number of inputs; m_2: number of non-radial inputs; NR indicates non-radial

Output Inefficiency

= Output Radial Inefficiency + Output Non-radial Inefficiency

Output Radial Inefficiency(for Hybrid model)

$$= \frac{s_1}{s}(\phi-1)$$

s: number of outputs; s_1: number of radial outputs

Output Non-radial Inefficiency(for Hybrid model)

$$= \frac{1}{s}\sum_{r=1}^{s_2} s_r^{NR+} / y_{ro}^{NR}$$

s: number of outputs; s_2: number of non-radial outputs; NR indicates non-radial

Benchmark (λ)

It contains the benchmark DMUs for the evaluated DMU with its λ. In other words, they are those DMUs in the reference set whose λ isn't zero.

Times as a benchmark for another DMU

It provides useful information for efficient DMUs (score = 1). If "Times as a benchmark for another DMU" is zero for an efficient DMU, it means that it is just efficient in default. There are no other DMUs take it as a benchmark, in other words, it is a lonely DMU with a special situation in terms of input and outputs. The more times of an efficient DMU as a benchmark for other DMUs, the more significant the benchmark is.

Σλ

It is the sum of the value of λ.

RTS Estimation

See 3.3.1.1.1 Returns to scale (RTS)

Original

The original value of inputs/outputs.

Radial Movement

Only available for radial and hybrid models. It is the radial part of improvement of inputs/outputs, the proportional decrease of inputs or the proportional increase of outputs. Positive values mean increase, and negative values mean decrease.

Slack Movement

It's absolute value is the s^- (input slack) or s^+ (output slack) in the LP equations. Positive values indicate increase, and negative values indicate decrease.

Projection

It is the efficient target.

For radial models, Projection =Original + Radial Movement + Slack Movement

For SBM models, Projection =Original + Slack Movement

α, β and γ

The values of α, β and γ in the solution.

Dual Prices

See 3.3.1.2.3 Dual Prices and Sensitivity Analysis

RHS Lower Bound and RHS Upper Bound

See 3.3.1.2.3 Dual Prices and Sensitivity Analysis

3.3.1.2.2 Results For Malmquist Models

We try to give an explanation of the Malmquist model in an easy-to-understand way, with an example supposing that there are two periods, named period 1 (the "from" period)and period 2 (the "to" period).

Please note that the column names in the results for Cost, Revenue, Profit and Revenue Cost Ratio Malmquist models are the same as those in the results for technical efficiency Malmquist Model. For example, in the results for Cost Malmquist Model, the Malmquist Index is named as "Malmquist Index", but in essence, it is "Cost Malmquist Index".

Efficiency(t)

It is the efficiency calculated with the current period as the reference set. For example, efficiency(2) is the efficiency in period 2 calculated using the input/output values in the same period –period 2 as the reference set.

Efficiency(t-1) for adjacent reference

It is the efficiency calculated with the previous period as the reference set. For example, efficiency(2-1) is the efficiency in period 2 calculated using the input/output values in period 1 as the reference set.

Efficiency(t+1) for adjacent reference

It is the efficiency calculated with the next period as the reference set. For example, efficiency(1+1) is the efficiency in period 1 calculated using the input/output values in period 2 as the reference set.

Efficiency_Fixed (t) for adjacent reference

It is the efficiency calculated with a user-defined fixed period as the reference set. For example, if the reference period is 1, the Efficiency_Fixed (2) is the efficiency in period 2 calculated using the input/output values in period 1 as the reference set.

Efficiency_Global (t) for global reference

It is the efficiency calculated with all periods as a whole as the reference set. For example, the Efficiency_Global (2) is the efficiency in period 2 calculated using the input/output values in period 1 and period 2 as the reference set.

Multiplicative measurement of productivity change

The multiplicative method measures the relative change of productivity. It is the commonly used method.

Efficiency Change

$$\text{Efficiency Change} = \frac{\text{Efficiency}(\text{t})\ of\ the\ 'to'\ period}{\text{Efficiency}(\text{t})\ of\ the\ 'from'\ period}$$

For example, Efficiency Change from period 1 to period 2 is

$$\text{Efficiency Change (1 TO 2)} = \frac{\text{Efficiency}\ (2)}{\text{Efficiency}\ (1)}$$

Malmquist Index

1) Adjacent reference:

$$\text{Adjacent Malmquist Index} = \left[\frac{\text{Efficiency}(\text{t-1})\ of\ the\ 'to'\ period}{\text{Efficiency}(\text{t})\ of\ the\ 'from'\ period} \times \frac{\text{Efficiency}(\text{t})\ of\ the\ 'to'\ period}{\text{Efficiency}(\text{t+1})\ of\ the\ 'from'\ period} \right]^{1/2}$$

$$\text{Adjacent Malmquist Index (1 TO 2)} = \left[\frac{\text{Efficiency}(\text{2-1})}{\text{Efficiency}(1)} \times \frac{\text{Efficiency}(2)}{\text{Efficiency}(\text{1+1})} \right]^{1/2}$$

2) Fixed reference:

$$\text{Fixed Malmquist Index} = \frac{\text{Efficiency\_Fixed}(\text{t})\ of\ the\ 'to'\ period}{\text{Efficiency\_Fixed}(\text{t})\ of\ the\ 'from'\ period}$$

$$\text{Fixed Malmquist Index (1 TO 2)} = \frac{\text{Efficiency\_Fixed}(2)}{\text{Efficiency\_Fixed}(1)}$$

3) Global reference:

$$\text{Global Malmquist Index} = \frac{\text{Efficiency\_Global}(\text{t})\ of\ the\ 'to'\ period}{\text{Efficiency\_Global}(\text{t})\ of\ the\ 'from'\ period}$$

$$\text{Global Malmquist Index (1 TO 2)} = \frac{\text{Efficiency\_Global}(2)}{\text{Efficiency\_Global}(1)}$$

Technological Change

$$\text{Technological Change} = \frac{\text{Malmquist Index}}{\text{Efficiency Change}}$$

Note: Malmquist Indices, Efficiency Change and Technological Change with fixed or global reference are circular. For example,

Fixed MI (1 TO 2) * Fixed MI (2 TO 3) = Fixed MI (1 TO 3).

In addition, Global Malmquist model with non-CRS technology doesn't suffer from the infeasibility problem.

Relationship between Malmquist results

- Efficiency change (CRS)* Technological Change (CRS) = Malmquist Index (CRS)
- Efficiency change (VRS)* Technological Change (VRS) = Malmquist Index (VRS)
- Malmquist Index (CRS) = Malmquist Index (VRS) * SEMI
- Malmquist Index (CRS) = Malmquist Index (VRS) * SEEC* SETC

See also 3.3.1.1.2 RTS

Additive measurement of productivity change

The additive method measures the absolute change of productivity.

Efficiency Change (Additive method)

$$\text{Efficiency Change} = \text{Efficiency}(\text{t})\ of\ the\ 'to'\ period - \text{Efficiency}(\text{t})\ of\ the\ 'from'\ period$$

For example, Efficiency Change from period 1 to period 2 is

$$\text{Efficiency Change (1 TO 2)} = \text{Efficiency}\ (2) - \text{Efficiency}\ (1)$$

Malmquist Index (Additive method)

4) Adjacent reference:

$$\text{Adjacent Malmquist Index} = \tfrac{1}{2} \times$$
$$\{[\text{Efficiency}(\text{t-1})\ of\ the\ 'to'\ period - \text{Efficiency}(\text{t})\ of\ the\ 'from'\ period] +$$
$$[\text{Efficiency}(\text{t})\ of\ the\ 'to'\ period - \text{Efficiency}(\text{t+1})\ of\ the\ 'from'\ period]\}$$

$$\text{Adjacent Malmquist Index (1 TO 2)} = \tfrac{1}{2} \times$$
$$\{[\text{Efficiency}(2\text{-}1) - \text{Efficiency}(1)] +$$
$$[\text{Efficiency}(2) - \text{Efficiency}(1\text{+}1)]\}$$

5) Fixed reference:

$$\text{Fixed Malmquist Index} =$$
$$\text{Efficiency\_Fixed}(\text{t})\ of\ the\ 'to'\ period - \text{Efficiency\_Fixed}(\text{t})\ of\ the\ 'from'\ period$$

$$\text{Fixed Malmquist Index (1 TO 2)} = \text{Efficiency\_Fixed}(2) - \text{Efficiency\_Fixed}(1)$$

6) Global reference:

$$\text{Global Malmquist Index} = \text{Efficiency\_Global}(\text{t})\ of\ the\ 'to'\ period - \text{Efficiency\_Global}(\text{t})\ of\ the\ 'from'\ period$$

$$\text{Global Malmquist Index (1 TO 2)} = \text{Efficiency\_Global}(2) - \text{Efficiency\_Global}(1)$$

Technological Change (Additive method)

$$\text{Technological Change} = \text{Malmquist Index} - \text{Efficiency Change}$$

Note: Malmquist Indices, Efficiency Change and Technological Change with fixed or global reference are circular. For example (Additive method),

Fixed MI (1 TO 2) + Fixed MI (2 TO 3) = Fixed MI (1 TO 3).

In addition, Global Malmquist model with non-CRS technology doesn't suffer from the infeasibility problem.

Relationship between Malmquist results (Additive method)

- Efficiency change (CRS) + Technological Change (CRS) = Malmquist Index (CRS)
- Efficiency change (VRS) + Technological Change (VRS) = Malmquist Index (VRS)
- Malmquist Index (CRS) = Malmquist Index (VRS) + SEMI
- Malmquist Index (CRS) = Malmquist Index (VRS) + SEEC + SETC

See also 3.3.1.1.2 RTS

3.3.1.2.3 Results For Network And Dynamic Models

For network models, there is another table to report the results for each node of every DMU. For dynamic models, there is another table to report the results for each period of every DMU.

3.3.1.2.3 Dual Prices and Sensitivity Analysis

The dual price (also named as shadow price, dual value) of an input (output) indicate that the objective function will change with the value of the dual price if the value of the right-hand side (RHS) for the input (output) is changed with 1 unit. Note that the sign indicates whether the objective function will increase or decrease.

The dual price remains constant as long as the RHS for the input (output) stays within the lower/upper bounds. If there is no dual price, or no lower/upper limit, then these values are (-)infinity[1].

The absolute values of the dual prices correspond to the values of the weights of inputs (outputs) in the dual model (Multiplier Model). Note that there often exist multiple optimal solutions for multiplier model, so the absolute values of the dual prices got from envelopment model might not be equal to the values of the weights got from corresponding multiplier model.

Caution: The sensitivity analysis for dual prices cannot be interpreted as the sensitivity of the weights of inputs (outputs) to the change of values of inputs (outputs) in DEA model.

[1] 1E+30 indicates +finite, and -1E+30 indicates -finite.

3.3.1.3 Options

There are some options for envelopment models in the Options tab:

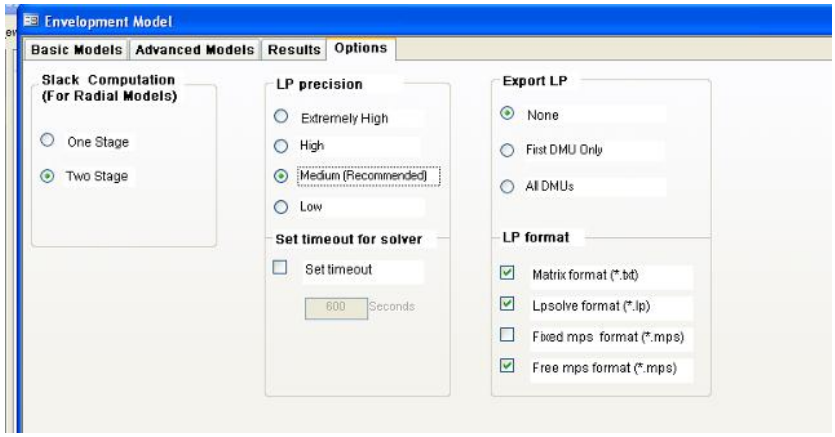


Figure 3-6 Options for Envelopment model

LP Precision

It sets the precision of the linear programming. In most cases, you needn't change this option, just keep the recommended option (medium). Only in case of unreasonable results found such as zero score with all slacks being zero, you can have a try by changing this option to higher level or lower level.

Note: One solution to deal with zeros in inputs/outputs is replacing zeros with a very small numbers, such as 0.01. If zeros have been replaced with too small numbers, such as 0.00000001, and meanwhile other values of inputs/outputs are relatively large numbers, such as more than 1,000,000, it will lead to the precision problem of LP solver under the default option of "LP Precision", i.e. medium. As a result, the problem of "LP numerical failure" or "LP infeasible" may occur. To avoid such a problem, you can 1) replace zeros with not-too-small numbers; or 2) set "LP Precision" to be "High" or "Extremely High".

Result Decimals

It sets the decimals of results.

Two Stage for Slack Computation

You can choose a second stage calculation for slacks in radial models by using this option.

Take CCR model as an example,

A two stage model carries out the LP with the following objective

$\min\ \theta - \varepsilon(es^- + es^+)$,

1st stage: $\min\ \theta$,

2nd stage: $\max es^- + es^+$,

while one stage model only carries out the 1st stage model.

No optima

Using this option, you can decide whether the score will be set to be 1 in case of no optima. The problem of no optima often takes place in Super-efficiency, Malmquist, and variable-benchmark and fixed-benchmark models.

Note: No optima may be infeasible, unbounded, degenerative or numerical failure.

Export LP

Linear programming equations of DEA models can be exported to text files with three formats: matrix format (*.txt), mps format (*.mps) and lp format(*.lp). It is very useful to help DEA users to develop their own DEA models using a general LP software.

LP equations are the core of DEA models, exporting LP equations is also helpful to verify the results of DEA models.

The user can choose to export LP for the first DMU only, or for all the DMUs, or not to export LP. In addition to LP equations, value of the objective function and values of all variables in the optimal solution are also exported. (See details in *MaxDEA Linear Programming Manual*.)

Set timeout for solver

It sets a timeout in seconds for the LP solver. The solve may not last longer than this time or it will return with a timeout.

3.3.2 Multiplier Model

3.3.2.1 Model Specifications

3.3.2.1.1 Basic Specifications(Basic Model)

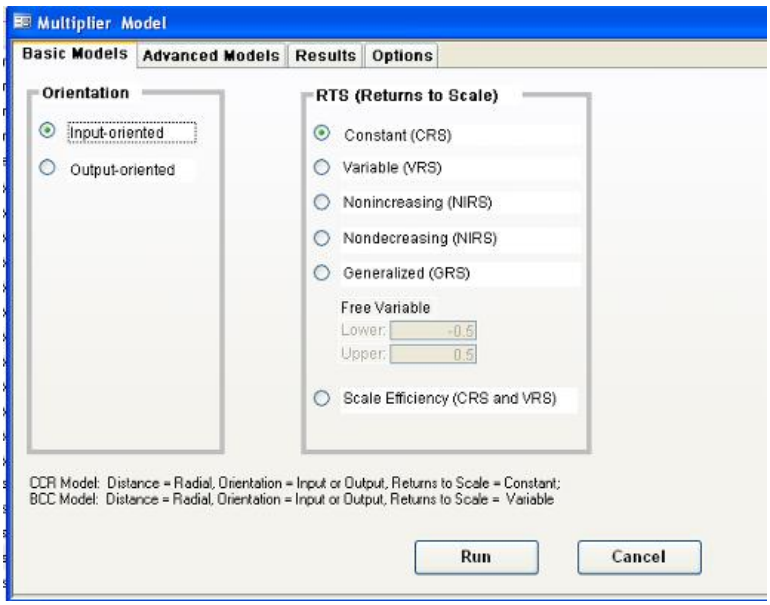


Figure 3-7(A) Basic specifications for Multiplier model

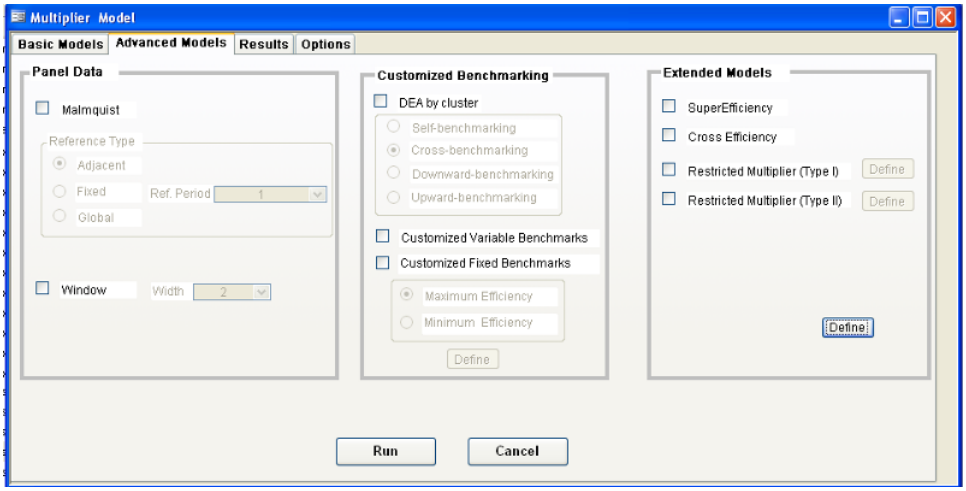


Figure 3-7(B) Advanced specifications for Multiplier model

The specifications of multiplier models include basic specifications (Orientation and RTS)

and advanced specifications as well, but much simpler than envelopment models .

There are two types of orientation available to multiplier models: input- and output-orientation. RTS is similar to envelopment models, with different additional options for GRS.

3.3.2.1.2 Advanced Specifications (Advanced Model)

Cross Efficiency Model

Mean cross efficiency is computed in addition to routine efficiency. And if the number of DMUs is not more than 252, the cross efficiency matrix will be shown in the results.

Note: there usually exist multiple optimal solutions for multiplier models, as a result, the cross efficiency score of a DMU is usually not unique. Using the "Two Stage" method to computing input and output weights can reduce the probability of multiple solutions dramatically(see 3.3.2.3).

Assurance Region Model

See "Restricted Multiplier Model"

Trade-offs between Inputs and Outputs

See "Restricted Multiplier Model"

Refer to :

Podinovski VV. Production trade-offs and weight restrictions in data envelopment analysis. J Oper Res Soc. 2004; 55(12): 1311-22

Restricted Multiplier Model

There are two types of restricted multiplier models available in MaxDEA, named as type I and type II respectively. The restricted multiplier model is to add constraints on the ratios or proportions of weights for inputs/outputs. It is also called Assurance Region model in

literature.

Type I restricted multiplier model is to add constraints on the ratios of weights for inputs/outputs, such as

$$L_1 \le \frac{\mu_2}{\mu_1} \le U_1$$

$$L_2 \le \frac{\mu_3}{\mu_1} \le U_2$$

while Type II restricted multiplier model is to add constraints on the proportion of a (some or all) weighted (virtual) input/output over the total weighted (virtual) inputs/outputs, for example, suppose there are two inputs and two outputs, we can restrict their virtual proportions as follows,

$$0.1 \le \frac{v_1 x_1}{v_1 x_1 + v_2 x_2} \le 0.3,$$

$$0.1 \le \frac{\mu_1 y_1}{\mu_1 y_1 + \mu_2 y_2} \le 0.3,$$

$$0.5 \le \frac{\mu_2 y_2}{\mu_1 y_1 + \mu_2 y_2} \le 0.8.$$

Please note that in type II restricted multiplier model, (1) the sum of the lower bounds of weight proportions must be less than or equal to 1, and (2) the sum of the upper bounds must be greater than or equal to 1 in case that all the weight proportions of inputs/outputs are restricted such as the above example, in which all the weight proportions of outputs are restricted.

To run the restricted multiplier model, set the restrictions first by a click on the “Define” button on the right side.

If you want to eliminate one piece of restriction, (1)click the arrow on the left, and (2) press the “Delete” key or use the “delete record button” on the toolbar. You can also disable the restriction through the active check box.

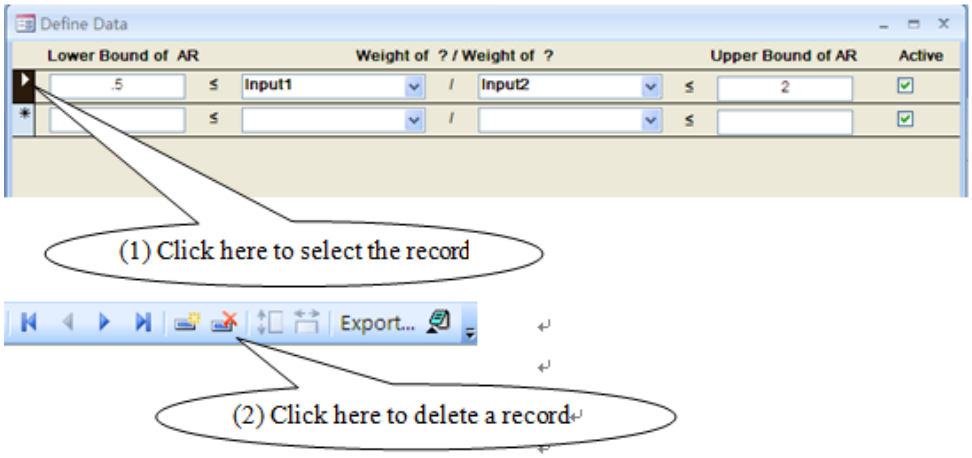


Figure 3-7-C Edit restrictions for Restricted Multiplier Model

Variable-benchmark Model

See “Customized Benchmarking”

Fixed-benchmark Model

See “Customized Benchmarking”

Minimum Efficiency Model

See “Customized Benchmarking”

Customized Benchmarking

In customized benchmarking model, you can customize the reference DMU set for calculating efficiency scores. Take the input-oriented CCR model as an example,

$$\max \theta = \mu' y_0$$

$$st \quad \nu' x_0 = 1$$

$$\mu' \mathrm{Y}^{\mathrm{r}} - \nu' \mathrm{X}^{\mathrm{r}} \le 0$$

$$\mu, \nu \ge 0 \ (\text{or } \mu, \nu \ge \varepsilon)$$

The superscript r indicates the DMUs included in the customized reference DMU set.

The above model is also called **variable-benchmark model** (Zhu J, 2009), because any of the DMUs in R might be the benchmark for a DMU evaluation, but it is not necessary for all the DMUs in R to be the benchmarks.

If you want one or some DMUs in R to be fixed benchmarks, you can develop a **fixed-benchmark model** (Zhu J, 2009) as follows, also take the input-oriented CCR model as an example,

$$\max \theta = \mu' y_0$$

$$st \quad \nu' x_0 = 1$$

$$\mu' \mathrm{Y}^{\mathrm{f}} - \nu' \mathrm{X}^{\mathrm{f}} = 0$$

$$\mu' \mathrm{Y}^{\mathrm{n}} - \nu' \mathrm{X}^{\mathrm{n}} \le 0$$

$$\mu, \nu \ge 0 \ (\text{or } \mu, \nu \ge \varepsilon)$$

f indicats the fixed benchmarks, and n indicates the non-fixed (variable) benchmarks.

If change the objective function of the fixed-benchmark model

$$\max \theta = \mu' y_0,$$

to

$$\min \theta = \mu' y_0,$$

The model will change from a maximum efficiency fixed-benchmark model to a **minimum efficiency** fixed-benchmark model (Zhu J, 2009). Such a change for output-oriented models is vice versa.

Variable-benchmarking can be applied to both envelopment models and multiplier

models, but fixed-benchmarking can be applied to multiplier models only.

To set a variable-benchmark model, open the menu “Run Multiplier Model”, check the option “Variable-benchmark”, and click the “Define” button to select variable benchmarks.

To set a fixed-benchmark model, check the option “Fixed-benchmark”, choose maximum efficiency or minimum efficiency, and click the “Define” button to select both variable benchmarks and fixed benchmarks. The selection of variable benchmarks is optional.

Note: Be cautious about setting the fixed benchmarks. If the fixed benchmarks are not properly chosen, it will result in infeasible LP.

3.3.2.2 Result Specifications

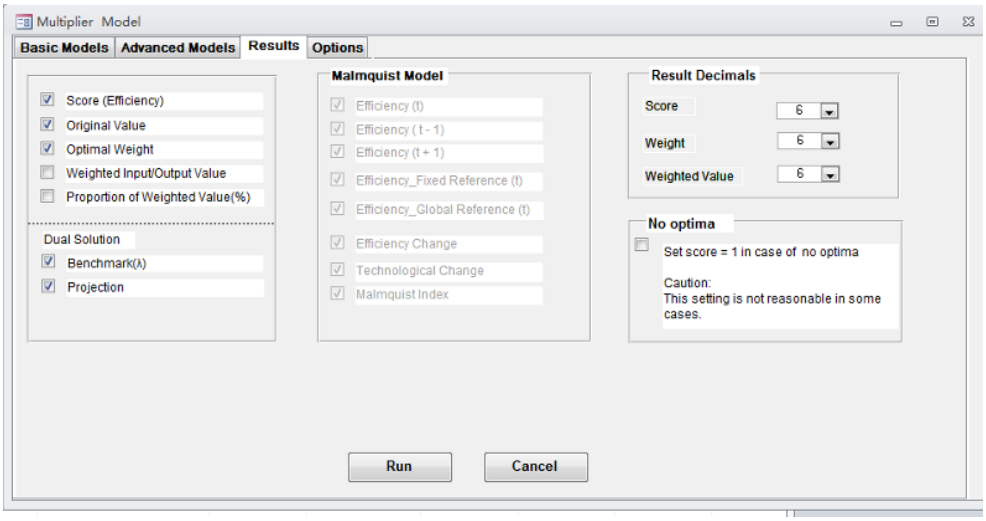


Figure 3-8 Results for multiplier model

The result specifications are simpler than envelopment models, so no further explains are given here.

Coefficient Lower Bound and Coefficient Upper Bound

See Sensitivity of Objective Function

Sensitivity of Objective Function

It provides the lower and upper bounds of a coefficient in the objective without causing any of the optimal values of the decision variables (weights of inputs and outputs) to change.

Caution: The sensitivity analysis for objective function cannot be interpreted as the sensitivity of the weights of inputs (outputs) to the change of values of inputs (outputs) in DEA model.

Dual Solution

It provides the benchmarks with the values of lambda and the values of projection.

3.3.2.3 Other options

This section is similar with envelopment model. The difference is that it has an option for "weight computation" in place of "slack computation", and has an additional specification for ε.

Two Stage for Weight Computation

Using the two stage method, the weights will be re-computed in the second stage, with the objective of minimizing (or maximizing) the average efficiency of other DMUs. This method is especially useful for computing cross efficiencies since there usually exist multiple optimal solutions in multiplier models.

ε Value

ε is set to be 0 in default. Be cautious to change this parameter. If it is not properly set, infeasible LP may occur.

Set timeout for solver

It sets a timeout in seconds for the LP solver. The solve may not last longer than this time or it will return with a timeout.

3.4 Export Results

Menu：Results - Browse Results / Export Results to Excel

Export results to Excel

After running the model, the results will show automatically, and you can browse or export the results to Excel at anytime.

Copy all the results to clipboard

Click the left top of the results window, or use “Ctrl + A” to select all the results. Then use “Ctrl + C” and “Ctrl + V” to copy and paste the results. You can also use the copy and paste button in the toolbar.

Qry_Results of Envelopment Model

NO	DMU	Score	Peer(λ)	Times as
1	A	.46078724	B(0.66666667)	0
2	B	1	B(1.00000000)	11
3	C	.82943284	B(1.06666667)	0
4	D	.82161172	B(1.20000000)	0
5	E	.42007291	B(0.62666667)	0
6	F	.61327678	B(1.53333333)	0
7	G	.7670922	B(1.46666667)	0
8	H	.53470841	B(1.01333333)	0
9	I	.62631417	B(1.26666667)	0
10	J	.66831228	B(1.66666667)	0
11	K	.56563812	B(1.73333333)	0

Figure 3-9 Copy all the results to clipboard

Copy part of the results to clipboard

Select one column：click the column title, and the column will be selected.

Select custom area：click the left top of the area you want to select, press “Shift” and click the right bottom of the area, then the area will be selected.

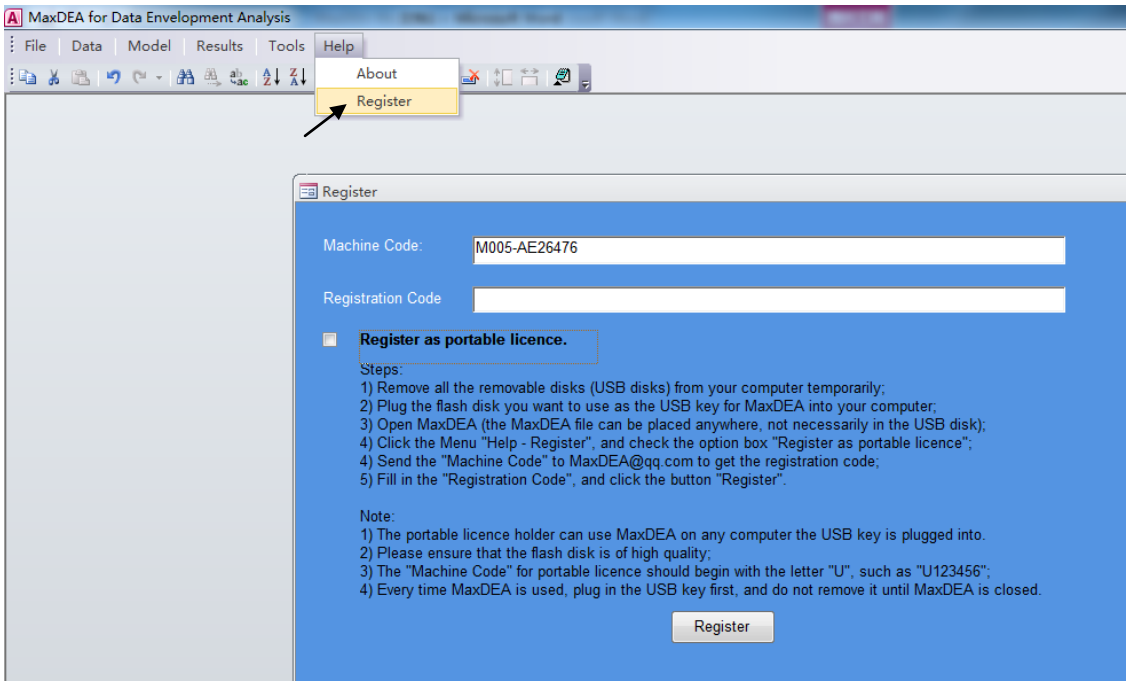
Qry_Results of Envelopment Model

NO	DMU	Score	Peer(λ)	Times as	Original
1	A	.46078724	B(0.66666667)	0	20
2	B	1	B(1.00000000)	11	19
3	C	.82943284	B(1.06666667)	0	25
4	D	.82161172	B(1.20000000)	0	27
5	E	.42007291	B(0.62666667)	0	22
6	F	.61327678	B(1.53333333)	0	55
7	G	.7670922	B(1.46666667)	0	33

Figure 3-10 Copy part of the results to clipboard

Chapter 4: Register MaxDEA Pro

MaxDEA Basic can be downloaded at **http://www.MaxDEA.cn** **free**. It is fully functional with the basic models (over 200 combinations). MaxDEA Basic will be upgraded to MaxDEA Pro after registration.



Steps for registration (fixed licence):

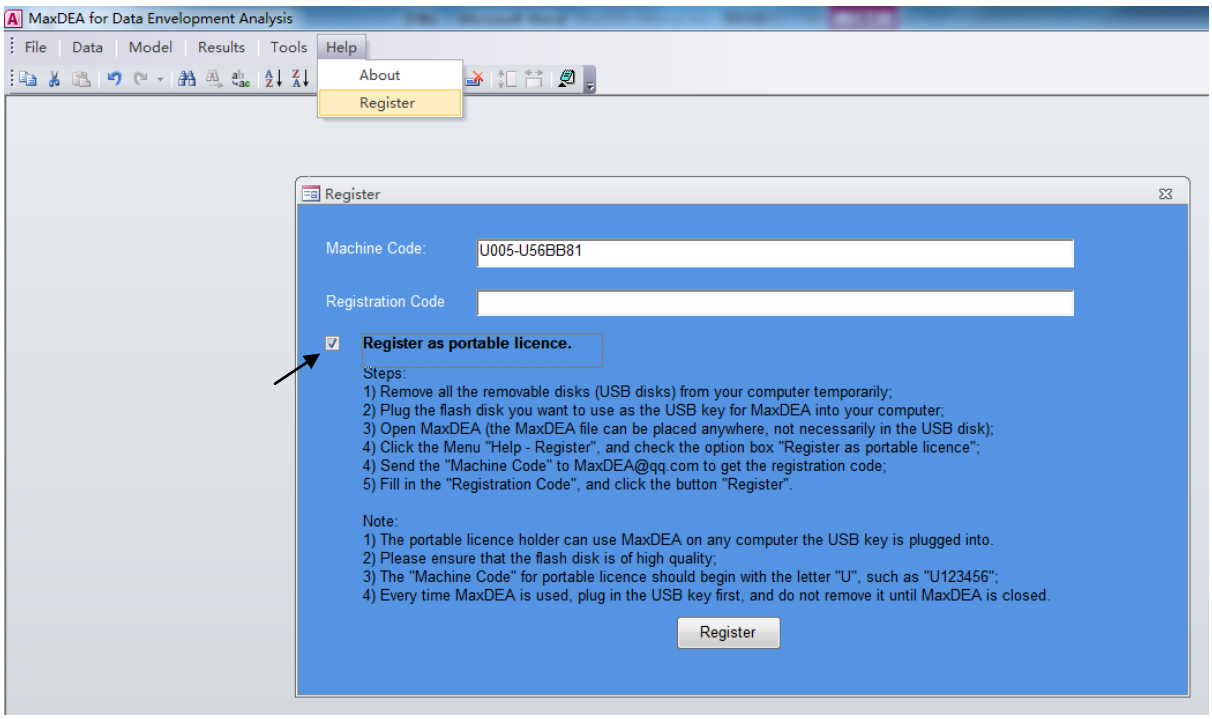
1) Buy a licence: Online payment at PayPal;

2) Download MaxDEA Basic from www.maxdea.cn;

3) Open MaxDEA, click the menu “Help – Register”, and send the “Machine Code” to MaxDEA@qq.com;

4) You will receive the registration code by email in 5 days (within 1 day in most cases);

5) Fill in the “Registration Code ” sent to you, click the button “Register”.



Steps for registration (portable licence):

1) Buy a licence: Online payment at PayPal;
2) Download MaxDEA Basic from www.maxdea.cn;
3) Remove all the removable disks (USB disks) from your computer temporarily;
4) Plug the flash disk you want to use as the USB key for MaxDEA into your computer;
5) Open MaxDEA (the MaxDEA file can be placed anywhere, not necessarily in the USB disk), click the Menu "Help - Register", remember to check the option box "**Register as portable licence**", and send the "Machine Code" to MaxDEA@qq.com;
6) You will receive the "Registration Code" by email in 5 days (within 1 day in most cases);
7) Fill in the "Registration Code", and click the button "Register".

Note:

1) The portable licence holder can use MaxDEA on any computer the USB key is plugged into.

2) The flash disk used for USB key is prepared by the user. Please ensure that the flash disk is of high quality;

3) The "Machine Code" for portable licence should begin with the letter "U", such as "U123456";

4) Every time MaxDEA is used, plug in the USB key first, and do not remove it until MaxDEA is closed.

As a policy of MaxDEA, update to a higher version is FREE of charge. For instance, MaxDEA Pro 5.0 can be updated to 6.0 or higher version free.

MaxDEA update is very convenient. Just download an updated version of MaxDEA Basic from http://www.MaxDEA.cn, and register it with your registration code.

Any problem, please contact MaxDEA@qq.com

Home page: http://www.MaxDEA.cn

Chapter 5 Frequently asked questions

1. **I cannot find the "setup" file in the downloaded zip file, why?**

   MaxDEA doesn't need installation. The only file needed to run MaxDEA is the mdb file (such as MaxDEA 4.mdb). All the data needed to develop a DEA model, including dataset and model options, are permanently stored in this file, which means that after closing and reopening the file, the database and model options are still there unchanged. MaxDEA provides extreme convenience to develop DEA models and keep backups of the models. If you want to keep a backup of a DEA model, just rename it as you want, such as Model_CCR.mdb, it is a full backup of the DEA model.

2. **Why I cannot open the program file (mdb file), why?**

   MaxDEA is developed with VBA for Access, so Microsoft **Access** is required for running MaxDEA.

   If you cannot open the program file (the file with extension "mdb" cannot be recognized), it indicates that Microsoft Office Access is not installed in your computer, and you must install MS Access 2003, 2007 or 2010 (Professional **OR** Runtime) first. Access **Runtime** can be downloaded **free** at Microsoft website.

3. **After registration, Can I use MaxDEA on multiple computers?**

   MaxDEA has two types of licences.

   One is fixed licence, which is bonded to a computer and can be used in this computer only. The licence will not be affected by reinstalling system, formatting hard disk, and even changing a new hard disk.

   The other is portable licence. This licence type uses a flash disk as the USB key. The license holder may work with MaxDEA on any computer the USB key is plugged into. The flash disk is prepared by the user.

4. **After reinstalling windows system, is the registration code still valid?**

   The licence is not affected by reinstalling system, formatting hard disk, and even changing a new hard disk.

5. **If the registered computer for MaxDEA fixed licence is scrapped, can I get a new registration code for a new computer?**

   With the fixed licence, if the registered computer is scrapped (the licence is not affected by reinstalling system), you can request A SECOND registration code for a new computer.

6. **If the USB disk for MaxDEA portable licence is damaged, can I get a new registration code for a new USB disk?**

   With the portable licence, MaxDEA can be used on any computer the USB key is plugged into.

   If the USB disk for MaxDEA portable licence is damaged, you can request A SECOND registration code for a new USB disk.

7. How to update MaxDEA to a higher version? Is update free?

As a policy of MaxDEA, update to a higher version is FREE of charge. For instance, MaxDEA Pro 5.0 can be updated to 6.0 or higher version free.

MaxDEA update is very convenient. Just download an updated version of MaxDEA Basic from http://www.MaxDEA.cn, and register it with your registration code.

8. After registration, how can I get technical support?

Technical support for MaxDEA is provided by email. If you have any problems on MaxDEA, please contact MaxDEA@qq.com. It is a free service.

9. How to backup my DEA models with MaxDEA?

MaxDEA provides extreme convenience to develop DEA models and keep backups of the models. If you want to keep a backup of a DEA model, just rename it as you want, such as Model_CCR.mdb, it is a full backup of the DEA model.

10. When I reopen MaxDEA, the data and model options are missing, why?

If you open MaxDEA directly in the zip file, the data may be lost after the program file is closed. It is strongly suggested that you unzip the file (MaxDEA.zip) first.

11. Can MaxDEA run multiple models at the same time? If yes, how to do it?

Multiple models can be run at the same time. Because the only files needed for running the program is MaxDEA.mdb, you can rename or copy this file freely. Each copy of this file contains one DEA model with all its data and model options saved in the file. And you can open and run multiple files simultaneously.

12. How to deal with zeros in inputs or outputs in MaxDEA?

There are many methods to deal with zero data in the literature. One easy approach is replacing zeros with a small number, such as 0.01. Please note that a too small number might cause a numerical problem in linear programming. Suppose, the value of input1 of DMU1 is 100000000, and the value of input1 of DMU2 is 0. If you replace the zero with 0.0000001, it will cause a error in solving the linear program.

13. What is the largest number of DMUs that MaxDEA can deal with?

Theoretically the number of DMUs that MaxDEA can deal with is unlimited. We have tested with a dataset with 20000 DMUs. Please note that the computing time will increase dramatically when the number of DMUs increases.

14. MaxDEA terminates with a message "Inputs must be numeric", why?

Please check your data. There may be spaces in the data, which will be treated as strings. This problems often take place when you copy the data from a pdf file.

Remove the spaces from the data, or do it in your original dataset and import the data again.

15. When I import data from Excel, MaxDEA terminates with a message "The search key was not found in any record " , why?

The first row of the Excel sheet contains column names, and there must not be spaces left to the column names. For example, column names such as “ Input1” will result in an error. But spaces in the middle of column names or right to the column names, such as “Input1 ” and “Input 1”, are permitted, but not recommended.

This problems often take place when you copy the data from a pdf file.

Remove the spaces from your data in Excel and import the data again.

In addition, this problem may also result from non-uniqueness of the column names. The column name must be unique for each column.

16. My data have 20 fields, but in the form of “Define Data” only the first 9 fields are displayed, why?

In any form, if your data are displayed incompletely, please scroll your mouse wheel or click the scroll bar on the right side to display the next contents.

Chapter 6: Reference books

MaxDEA is developed on the basis of the following (but not confined to the following) books on Data Envelopment Analysis.

Cooper W W, Seiford L M and Zhu J. Handbook on Data Envelopment Analysis,Springer (Kluwer Academic Publishers), Boston, 2004

Cooper W W，Seiford L M，Tone K. Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software. New York: Springer Science+Business Media, LLC, 2007

Zhu J. Quantitative Models for Performance Evaluation and Benchmarking: DEA with Spreadsheets, 2nd Edition, Springer, Boston, 2009